Memory

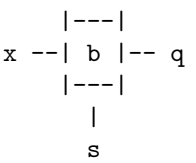
We consider ciruits that exhibit memory.

Functions have no memory

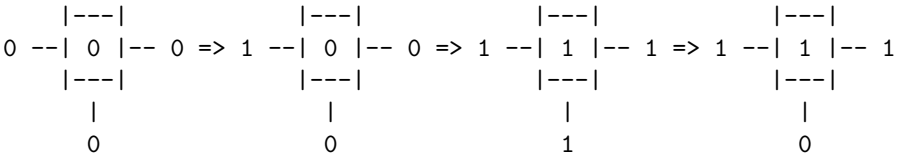
So far we have been describing circuits that compute Boolean functions. The circuits we have considered are combinational circuits, in which there is no path of wires and gates from the output of a gate back to its input. Such circuits are also called feedforward, in contrast to the feedback created by loops.

The output of a combinational circuit is strictly a function of the current inputs; it has no "memory" of past inputs. In addition, we need devices that "remember" or "store" a value for us, (somewhat) independently of the current values of the inputs.

In particular, we'd like a 1-bit memory that functions as follows. It has two inputs, which we call x and s, and one output q, together with some internal *state* b that stores a 0 or a 1. The input s functions as a "set" command; when $s = 0$, the output q is equal to the stored bit, regardless of the value of x. When $s = 1$, the value of x is copied into the internal state, and also appears on the output q. Our 1-bit memory element will be pictured as follows.



Some of its behavior can be illustrated by the following sequence of changes to its inputs.



In the first situation, the internal state is 0 and the output is 0 and two inputs are 0. Then the x input is changed to 1, and there is no change in the state or output. Then the s input is changed to 1, and the state and output are changed to the value of x, that is 1. Finally, the s input is changed to 0, and the state and output remain 1. Comparing the second with the fourth situations, the x input is 1 and the s input is 0 in both cases, but the outputs are different. This illustrates that this circuit is not simply a function of its current inputs (x and s), but "remembers" the value of x at the last time s was 1.

Digression: What was core memory?

A number of technologies have been used to implement memory. One that was quite important in the 1960's and 1970's and isn't much used anymore is core memory. When the random access memory of a computer is referred to as "core," it reflects the former dominance of this memory technology. (This usage is preserved in the name "core" for the annoying large file that may be created in your directory when an application crashes; it is short for "core dump," a snapshot of the contents of random access memory intended to help you track down the causes of the mysterious crash.)

The basic physical element is a little ferrite core (doughnut-shape) with an electric wire passing through it. When sufficient electric current is passed through the core in one direction, the core is magnetized in one direction, and retains that magnetization until sufficient electric current is passed through the core in the other direction, magnetizing it in the other direction. This behavior allows us to store a value of 0 or 1 by which way the core is magnetized, but a write-only memory is not much use; we also need a way to read the value.

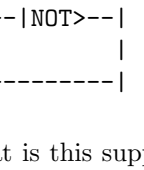
For that purpose, another wire (the sense wire) is threaded through the core, and then sufficient current is passed through the core on the set wire in the 0 direction to write it. If the core was holding the 1 value, then a pulse can be detected on the sense wire induced by the change in the magnetic field. If the core was holding the 0 value, the magnetic field does not change, and no pulse is detected.

Thus, the 0 or 1 value held in the core can be read, at the cost of forcing the core to the 0 state, and losing the previous information it held. This is an example of a "destructive read." To overcome this, the memory circuitry was designed to write back the value read, thus implementing a "nondestructive read."

A private set wire for every bit of memory (the sense wire could be shared) was not a practical design, so most accounts you'll see of core memory emphasize that the cores were arranged in rectangular arrays, with two set wires (one horizontal and one vertical) passing through each core. Then by sending half the "sufficient" current through a vertical wire and half through a horizontal wire only the single core through which both wires passed would be affected by the set signal. For example, in a 32 by 32 array of 1024 bits, this idea cuts the number of different set wires from 1024 (one per bit) to 64 (32 horizontal, 32 vertical.)

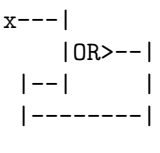
Sequential circuits and memory

Sequential circuits have feedback loops; there is a path of wires from the output of some gate back to the input of that gate. Here is a simple example:



What is this supposed to mean? The picture represents a NOT gate with its output fed back as its input. It is not clear how to reason about this configuration; if the input is 0, then the output is 1, so the input is 1 and the output is 0, so the input is 0, etc. It seems to be contradictory: the output is 0 if and only if the output is 1. The physical reality is that there is a time delay before the output changes, so this kind of configuration will exhibit an oscillatory behavior, with the value on the wire changing rapidly between 0 and 1. This kind of behavior is in fact useful, as it provides a regularly spaced timing signal that can be used to control the time-order of events in hardware.

Here is another simple example:



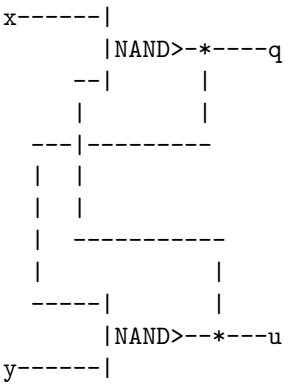
This is intended to depict an OR gate with one input x and the output fed back as the other input to the OR gate. Suppose we start with $x = 0$ and the output of the OR gate 0. This is a consistent and stable state of the circuit, because the value computed by the OR gate is 0 when its two inputs are 0. Now, if we set $x = 1$, the output will change to 1 and this is also a consistent and stable state of the circuit, since both

inputs to the OR gate are 1 and the output is 1. However, if we now set $x = 0$, the output of the circuit remains 1, because the inputs to the OR are 1 and 0, so the output is 1, another stable state of the circuit. Clearly, the output remains 1 indefinitely, independent of the value of x. This circuit exhibits a rudimentary kind of memory; it remembers forever whether the input x has ever been set to 1.

To implement the 1-bit memory element we seek, we'll need a more complicated sequential circuit. Recall the Boolean functions NAND and NOR: (x NAND y) is 0 if both x and y are 1, and 1 otherwise, while (x NOR y) is 1 if both inputs are 0 and 0 otherwise. NAND is a complete basis for the Boolean functions, as is NOR.

A more complex sequential circuit

We now consider two NAND gates, with the output of each fed back to be one input of the other. In ASCII diagram, we have the following.



The inputs to this circuit are x and y, and the outputs are q and u. There are loops in this circuit – if we trace the output wire of the first NAND gate around to the input of the second NAND gate, to the output of the second NAND gate, to the input of the first NAND gate, to the output of the first NAND gate, we are back where we started: a loop. Thus, this circuit is sequential.

One way to figure out what is happening in this circuit is to consider the equations:

$$q = (x \cdot u)'$$

and

$$u = (y \cdot q)'.$$

A solution of these equations consists of an assignment of 0's and 1's to the variables x, y, q, and u such that both equations are true; this represents a possible stable configuration of the circuit. There are 16 possible combinations to test, and we find that there are just 5 combinations that give a solution for both equations:

x	y	q	u
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	0
1	1	0	1

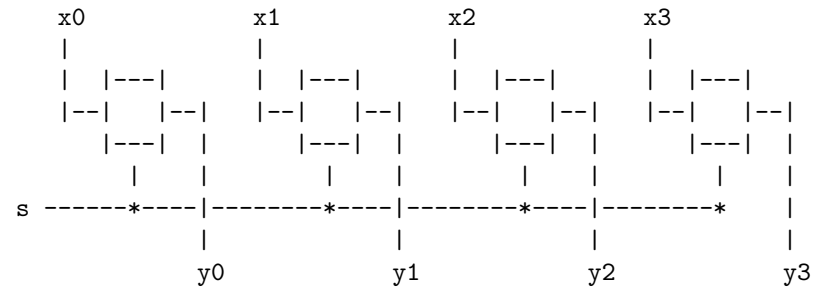
The last two lines show that neither q nor u is a function of the values of x and y, because we can have $x = y = 1$ and $q = 1$ or $q = 0$, and similarly for u. The values of q and u are not uniquely determined by the values of x and y; the circuit has state, or memory.

We exploit the state to store information as follows. We AVOID setting x and y to 0 simultaneously, and thereby avoid the first line of the table. Then there are two states of the circuit: where $q = 1$ and $u = 0$, and where $q = 0$ and $u = 1$. If the inputs are $x = 1$ and $y = 1$, the circuit stays in whichever state it is currently in. If the inputs are $x = 0$ and $y = 1$, the circuit goes from whichever state it is currently in to the state where $q = 1$ and $u = 0$. If the inputs are $x = 1$ and $y = 0$, the circuit goes from its current state to the state where $q = 0$ and $u = 1$.

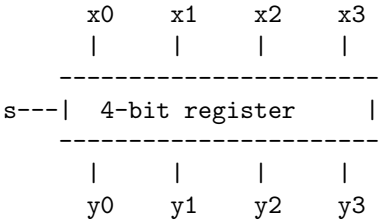
This use of the circuit allows us to store one bit of information. That is, $q = 1$ signifies that the last time the inputs were not both 1, it was x that was 0. Correspondingly, $q = 0$ signifies that the last time the inputs were not both 1, it was y that was 0. A further elaboration of this circuit gives an implementation of our desired 1-bit memory.

Registers

We consider possible uses of a 1-bit memory element. A register is a device that can store some fixed number of bits in memory. For example, suppose we connect four 1-bit memories with a common set line as follows.



Then as long as `s` is 0, the outputs `y0`, `y1`, `y2`, and `y3` are equal to the bits stored in the four 1-bit memories, respectively. When `s` is set to 1, the values of the inputs `x0`, `x1`, `x2`, and `x3` are copied into the corresponding 1-bit memories and also appear on the corresponding output wires. This allows us to read and set the four bits in the register. Larger or smaller registers can be constructed by similarly connected larger or smaller numbers of 1-bit memories. We abstract this circuit and picture it as a rectangle.



Note that both registers and combinational circuits are depicted with rectangles, so some context is necessary to decide which kind of circuit is intended.