Computer Science 202a
Homework #9
Due in Prof. Angluin's mailbox (1st floor, AKW)
By noon, Wednesday, Dec. 13, 2006

Please include how much time you have spent on (1) reading and (2) doing the problems for this assignment; this is worth 2 points of extra credit.

1. Counterfeit coin problem. Text 10.2, problem 10, page 708.

2. Depth first spanning tree. Text 10.4, problem 14, page 735.

3. Breadth first spanning tree. Text 10.4, problem 16, page 735. Please solve this only for the tree in problem 14.

4. Solve the following linear system using Gaussian elimination and back substitution. Please show the results after each downsweep.

$$\begin{aligned} 3x + 4y + z &= 12 \\ 6x + 9y + 3z &= 27 \\ 3x + y + 2z &= 11 \end{aligned}$$

5. Solve the following linear system using Gaussian elimination and back substitution. Please show the results after each downsweep.

$$\begin{aligned} 3x + 4y + z &= 12 \\ 6x + 9y + 3z &= 27 \\ 3x + 5y + 2z &= 11 \end{aligned}$$

6. Find a basis for the space spanned by the following three vectors:

$$(3, 4, 1), (6, 9, 3), (3, 5, 2).$$

Justify your claim that it is a basis.

7. Does the following greedy method produce a basis for the space spanned by a finite set of vectors $\{v_1, v_2, \ldots, v_k\}$? Justify your answer.

Let $B_0 = \emptyset$. For each $i = 1$ to k, if $B_{i-1} \cup \{v_i\}$ is linearly independent, then let $B_i = B_{i-1} \cup \{v_i\}$; otherwise, let $B_i = B_{i-1}$. The output is B_k.

8. Can the following pairs of matrices exist? Justify your answers.

   (a) A 2×3 matrix A and a 3×2 matrix B such that $A \cdot B$ is the 2×2 identity matrix.

   (b) A 3×2 matrix C and a 2×3 matrix D such that $C \cdot D$ is the 3×3 identity matrix.

9. Let n be a positive integer, and I be the $n \times n$ identity matrix. An *elementary matrix* of dimension n is defined to be I, or I with two rows exchanged, or I with one 0 or one 1 changed to a nonzero entry.

   For each of the following row operations, give an elementary matrix E of dimension 3 such that if A is any 3×3 matrix, then $E \cdot A$ is the result of applying the given row operation to A. Also give the inverse of E in each case.

   (a) Subtract twice the first row from the second.

   (b) Interchange the second and third rows.

   (c) Multiply the third row by 6.

10. Show that the matrix A can be written as $L \cdot U$, where L is a product of elementary matrices, and A is the matrix

$$\begin{pmatrix} 3 & 4 & 1 \\ 6 & 9 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

   and U is the matrix

$$\begin{pmatrix} 3 & 4 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

   Hint: $A = E^{-1} \cdot E \cdot A$ if E is an invertible matrix.