

YOUR NAME PLEASE:

Computer Science 202a
PRACTICE Midterm
October 16, 2006

Closed book. No electronic devices. You may use one double-sided 8.5 x 11 inch sheet of notes prepared by you. I will provide you with a copy of the two inside front cover pages of the text during the test.

Show ALL work you want graded ON THE TEST ITSELF, including the backs of pages as necessary.

For problems that do not ask you to justify the answer, an answer alone is sufficient. However, if the answer is wrong and no derivation or supporting reasoning is given, there will be no partial credit.

GOOD LUCK!

problem	points	actual
1	10	
2	20	
3	20	
4	20	
5	20	
6	10	
total	100	

1. (10 points) Give a truth table to show the logical equivalence of the following two Boolean formulas. Remember to state what property of your truth table shows their logical equivalence.

$$(p \rightarrow q) \vee (p \rightarrow r)$$

$$p \rightarrow (q \vee r)$$

2. $A \oplus B$ is the set of elements in A or B but not both.

(a) (10 points) Prove that for all sets A , B , and C ,

$$A \oplus (B \cup C) \subseteq (A \oplus B) \cup (A \oplus C).$$

(b) (10 points) Give an example of sets A , B , and C such that

$$A \oplus (B \cup C) \neq (A \oplus B) \cup (A \oplus C).$$

3. Consider the following five statements:

1. Students are eating, watching TV, or doing laundry.
2. Some students are not doing laundry.
3. No student who is alert is snoring.
4. Students who are eating are alert.
5. Some students are not snoring.

(a) (10 points) Specify a domain of discourse and some predicate symbols and their meanings, and use them to translate each of the five statements above into predicate logic.

(b) (10 points) Is statement (5) above a logical consequence of statements (1), (2), (3) and (4)? Please justify your answer.

4. Consider the following functions, specified with their domains and co-domains. For each, say whether it is one-to-one, onto, or both, or neither. Justify your answers. (Each part is worth 5 points.)

(a) $f(x) = x^2$, domain = \mathbf{Z} , co-domain = \mathbf{N} .

(b) $g(S) = \sum_{i \in S} i$, domain = $P(\{1, 2, 3\})$, co-domain = $\{0, 1, 2, 3, 4, 5, 6\}$. (Recall that an empty sum is equal to 0 by convention.)

(c) $h(x) = \lfloor x \rfloor + x$, with domain and co-domain both equal to the set of positive real numbers.

(d) $k(x) = x/2$ if x is even, and $k(x) = (3x + 1)/2$ if x is odd, with domain and co-domain equal to the set of positive integers.

5. How many? Justify your answers. (Each part is worth 5 points.)

(a) Functions from $\{1, 2, 3\}$ to $\{1, 2, 3, 4, 5\}$?

(b) One-to-one functions from $\{1, 2, 3\}$ to $\{1, 2, 3, 4, 5\}$?

(c) Different strings that are rearrangements of all 11 letters of MISSISSIPPI?

(d) Ways of choosing 5 bagels from 4 different kinds of bagels (assuming there is an unlimited supply of each kind of bagel)?

6. (10 points) Define the sequence a_n by $a_0 = 1$, $a_1 = 3$ and for all $n \geq 0$, $a_{n+2} = a_{n+1} - 2a_n$ if $2a_n < a_{n+1}$ and $a_{n+2} = a_{n+1} + a_n$ if $2a_n \geq a_{n+1}$. Give the first 10 terms of this sequence, find an expression for a_n in terms of n , and prove by mathematical induction that it is correct.