Problem Session 1

CPSC 202a

September 18, 2006

The Foundations: Logic and Proofs

1. Find a compound proposition involving the propositional variables p, q, r, and s that is true when exactly three of these variables are true and false otherwise.

2. Let $P(x)$ be the statement "student x knows calculus" and let $Q(y)$ be the statement "class y contains a student who knows calculus." Express each of these as quantifications of $P(x)$ and $Q(y)$

   - Some student knows calculus
   - Some students (plural) know calculus
   - Not every student knows calculus
   - Every class has a student who knows calculus
   - There is at least one class with no students who know calculus

3. Translate each of these nested quantifiers into an English statement that expresses a mathematical fact (the domain is Real numbers).

   - $\exists x \forall y(x + y = y)$
   - $\forall x \forall y(((x > 0) \wedge (y < 0)) \rightarrow (x - y > 0))$
   - $\exists x \exists y(((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$
   - $\forall x \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

4. Prove that the sum (and product) of two rational numbers is rational. You can assume that integers are closed under addition and multiplication.

5. Derive the law of the excluded middle, that $(p \vee -p)$ is a tautology; do not use De Morgan's laws.

6. Prove that $\sqrt{3}$ is irrational.[1]

[1] difficult