Solutions for Problem Session 2*

CPSC 202a

September 26, 2006

Basic Structures: Sets and Functions

1. If $S = \{a, b\}$ what is $P(P(S))$, the power set of the power set of S?

$$P(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$P(P(S)) = \{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\}, \{\emptyset, \{a\}\}, \{\emptyset, \{b\}\},$$
$$\{\emptyset, \{a, b\}\}, \{\{a\}, \{b\}\}, \{\{a\}, \{a, b\}\}, \{\{b\}, \{a, b\}\}, \{\emptyset, \{a\}, \{b\}\},$$
$$\{\emptyset, \{a\}, \{a, b\}\}, \{\emptyset, \{b\}, \{a, b\}\}, \{\{a\}, \{b\}, \{a, b\}\}, \{\emptyset, \{a\}, \{b\}, \{a, b\}\}\}$$

2. What can you say about sets A and B if we know:

   - $A \cup B = A$
     $B \subseteq A$
   - $A \cap B = A$
     $A \subseteq B$
   - $A - B = A$
     $A \cap B = \emptyset$
   - $A \cap B = B \cap A$
     always true; can't say anything
   - $A - B = B - A$
     $A = B$
   - $A \cup B = A \cap B$
     $A = B$

*some of these problems are variants of problems from the textbook

3. Let $A \oplus B$ be the set containing the elements in either A or B but not in both. Is $\oplus$ an associative operation? Justify your answer.

   Yes, it is associative. We need to show that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$. Assume x is not in any of these sets. Then it is trivially in neither of $(A \oplus B) \oplus C$ or $A \oplus (B \oplus C)$. Now assume x is in only one of these sets... (looking at all of the cases finishes the proof)

4. If f and $f \circ g$ are onto, is g onto?

   No. Suppose $A = \{a\}$, $B = \{b, c\}$, and $C = \{d\}$ If $g(a) = b$, $f(b) = d)$, and $f(c) = d$ then f and $f \circ g$ are onto, and g is not.

5. If x is a real number, and m and n are positive integers, for which values of m, n, and x is $\lfloor \frac{\lfloor x \rfloor + n}{m} \rfloor = \lfloor \frac{x+n}{m} \rfloor$?

   It is true for all values of m, n, and x in the domain. Let q be an integer, where $q = \lfloor x \rfloor$. Then $x = q + \epsilon$ for some $0 \leq \epsilon < 1$. Now we want to compare $\lfloor \frac{q+n}{m} \rfloor$ and $\lfloor \frac{q+\epsilon+n}{m} \rfloor$. Let v be $q + n$. v is an integer because both q and n are. We want to see if $\lfloor \frac{v}{m} \rfloor = \lfloor \frac{v+\epsilon}{m} \rfloor$. Since v and m are both positive integers, ϵ needs to be ≥ 1 for the two sides of the equation to be different. Since $\epsilon < 1$, we are done.

6. Let $S = \{x | x \notin x\}$ (S contains a set x if x does not belong to itself).

   - is S a member of S? (justify your answer)
     If S were a member of S then $S \in S$. Since $S = \{x | x \notin x\}$, It is not true that $S \in S$. Therefore $S \notin S$. Contradiction. Therefore, **No.**
   - is S not a member of S? (justify your answer)
     If $S \notin S$, since $S = \{x | x \notin x\}$ then $S \in S$. Contradiction. Therefore, **No.** [1]

[1]this is Russell's paradox. covered in class.