Problem Session 3

CPSC 202a

October 2, 2006

Functions and Cardinalities

1. If A and B are countable sets, is $A \times B$ necessarily also a countable set? Justify your answer.

2. Is the set of functions from the positive integers to the set $\{a, b\}$ countable? Justify your answer.[1]

Mathematical Induction

3. Prove that $\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1}+(-1)^n}{3 \cdot 2^n}$ when n is a non-negative integer.

4. What is wrong with this "proof" that all horses are are the same color?

   Let $P(n)$ be the proposition that all of the horses in a set of n horses are the same color.

   Basis step: Clearly, $P(1)$ is true.

   Inductive Step: Assume that $P(k)$ is true, so that all the horses in any set of k horses are the same color.

   Now, consider any $k+1$ horses; number these horses from 1 to $k+1$. The first k of these horses all must have the same color by the inductive hypothesis. The last k of these must also have the same color, by the inductive hypothesis. Because the set of the first k horses and the set of the last k horses overlap, all $k+1$ must be the same color. This shows that $P(k+1)$ is true and finishes the proof by induction.

5. Prove that any postage of ≥ 18 cents can be formed using just 4-cent stamps and 7-sent stamps.

6. Suppose that n teams play a round-robin tournament in which there are no tie games. Prove that, no matter what the individual game outcomes are, one can always number the teams $t_1, t_2, ..., t_n$ so that t_1 beat t_2, t_2 beat t_3, and so on through t_{n-1} beat t_n.

[1] difficult