Problem Session 6

CPSC 202a

November 6, 2006

Recurrence Relations

1. A person deposits \$1000 in an account that yields 9% interest compounded annually.
   - Set up a recurrence relation for the amount in the account at the end of n years.
   - Find an explicit formula for the amount in the account at the end of n years.
   - How much money will the account have after 100 years?

2. Let $L(n) = L(\frac{n}{2}) + 1$ where $n = 2^k$. Prove by induction that $L(n) = \log_2(n) + 3$

3. We have n dollars. Every day we buy one of the following products: pretzel (1 dollar), candy (2 dollars), ice cream (2 dollars). We continue until the n dollars have been spent. What is the number B_n of distinguishable ways of spending all the money?

4. Solve the simultaneous recurrence relations

$$a_n = 3a_{n-1} + 2b_{n-1}$$
$$b_n = a_{n-1} + 2b_{n-1}$$

   with $a_0 = 1$ and $b_0 = 2$.

5. In the tower of Hanoi puzzle, suppose our goal is to transfer all n disks from peg 1 to peg 3, but we cannot move a disk directly between pegs 1 and 3. (ie each move must involve disk 2). As usual, we cannot place a disk on top of a smaller disk. How many moves are required to solve the puzzle for n disks?

6. Suppose f satisfies the recurrence $f(n) = 2f(\sqrt{n}) + \log n$ when n is a perfect square greater than 1 and $f(2) = 1$.
   - What is $f(16)$?
   - Use Master theorem to find an estimate for $f(n)$. (Hint: substitute $m = \log n$)