YOUR NAME PLEASE:

Computer Science 202
Midterm
1-2:15 pm, 18 October 2016

Closed book and closed notes. No electronic devices. Show ALL work you want graded ON THE TEST ITSELF.

For problems that do not ask you to justify the answer, an answer alone is sufficient. However, if the answer is wrong and no derivation or supporting reasoning is given, there will be no partial credit.

Recall that $\mathbb{N}$ denotes the set of natural numbers (integers greater than or equal to zero) and $\mathbb{Z}$ denotes the set of all integers (positive, negative and zero).

GOOD LUCK!
1. (10 points) Give a succinct definition of each of the following concepts. Assume $A$, $B$ and $C$ are sets of positive integers, $f$ is a function with domain $A$ and co-domain $B$, and $g$ is a function with domain $B$ and co-domain $C$.

(a) The union of $A$ and $B$, denoted $A \cup B$.

(b) The function $f$ is injective.

(c) The set difference of $A$ and $B$, denoted $A \setminus B$.

(d) The power set of $A$, denoted $\mathcal{P}(A)$.

(e) The composition of $g$ and $f$, denoted $(g \circ f)$. 
2. (10 points) Determine whether the following statements are true for ALL sets of positive integers $A$, $B$, and $C$. Your answer should be a proof or a counterexample, NOT a Venn diagram.

(a) If $(A \setminus B) = (A \setminus C)$ and $(B \setminus A) = (C \setminus A)$, then $B \subseteq C$.

(b) If $\mathcal{P}(A) \subseteq \mathcal{P}(B) \cup \mathcal{P}(C)$ then $A \subseteq B \cap C$. 
3. (12 points) For each of the following functions from the positive integers to the positive integers, give a table of the values of the function for $1 \leq n \leq 6$. Then state whether the function is injective and whether it is surjective, and justify your answers.

(a) $f(n)$ is $n - 1$ if $n$ is even, or $2n$ if $n$ is odd.

(b) $g(n)$ is the largest element of the set $\{j \in \mathbb{Z} \mid 3^{j-1} \leq n\}$.
(Problem 3, continued)

(c) \( h(n) = \lceil (3n - 1)/2 \rceil \).

(d) \( k(n) \) is the number of positive integers that (evenly) divide \( n \).
4. (12 points) We recursively define the function \( g(n) \) for all \( n \in \mathbb{N} \) as follows. We set \( g(0) = 1 \) and \( g(1) = 3 \), and for all \( n \geq 2 \),

\[
g(n) = g(n-1) + g(n-2).
\]

Prove the following statement by strong mathematical induction, identifying the predicate \( P(n) \), the base case(s), and the inductive hypothesis.

\[
(\forall n \in \mathbb{N})(g(n) \geq (3/2)^n)
\]
5. We consider a domain that consists of all the animals in a certain zoo. We define the following predicates.

\( E(x) \) means \( x \) is an elephant
\( G(x) \) means \( x \) is a giraffe
\( M(x) \) means \( x \) is a monkey
\( H(x) \) means \( x \) is happy
\( N(x) \) means \( x \) is noisy
\( (x = y) \) means \( x \) is equal to \( y \)
\( R(x, y) \) means \( x \) respects \( y \)

(a) With these predicates write the following statements as logical formulas.

i. (2 points) At least one monkey is happy and noisy.

ii. (2 points) Every giraffe respects every elephant.

iii. (2 points) There is an elephant that respects itself and respects no other animal.
iv. (2 points) Every happy animal respects exactly those animals that respect all happy animals.

v. (2 points) Some animal respects no elephants.

(b) (4 points) Does the last statement (v) in part (a) follow logically from the other statements (i-iv)? Please justify your answer by giving an informal proof or a counterexample (that is, an example of a zoo making (i-iv) true and (v) false.)
6. (a) (4 points) Let \( f(n) \) and \( g(n) \) be functions with domain and co-domain \( \mathbb{N} \). Give the definition of the concept: \( f(n) \) is in \( \Omega(g(n)) \).

(b) (8 points) We define the function \( f(n) \) with domain and co-domain \( \mathbb{N} \) as follows. For every \( n \in \mathbb{N} \), if \( n \) is even then \( f(n) = n/2 \) and if \( n \) is odd, then \( f(n) = 3n^2 + 1 \). Prove that \( f(n) \) is in \( O(n^2) \), and that \( f(n) \) is not in \( \Omega(n^2) \).
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