

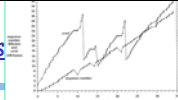
Transport Bandwidth Allocation

10/26/2009

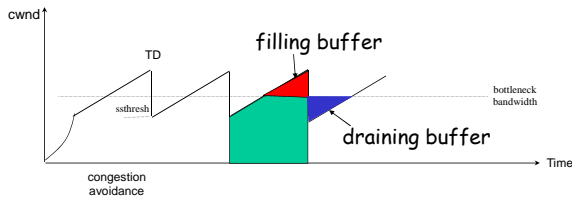
Admin.

- Questions on programming assignment 2?
- Date of exam 1?

Recap: TCP/Reno Queuing Dynamics



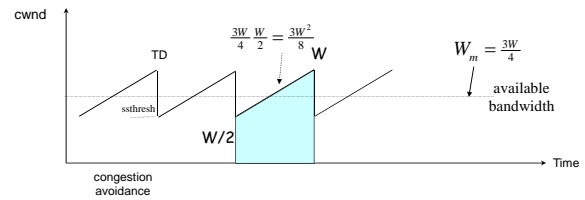
- Consider congestion avoidance only



There is a filling and draining of buffer process for each TCP flow.

Recap: TCP/Reno Throughput Modeling: Relating W with Loss Rate p

- Consider congestion avoidance only



Assume one packet loss (loss event) per cycle
 Total packets send per cycle = $(W/2 + W)/2 * W/2 = 3W^2/8$
 Thus $p = 1/(3W^2/8) = 8/(3W^2)$

$$W = \frac{\sqrt{8/3}}{\sqrt{p}} = \frac{1.6}{\sqrt{p}} \Rightarrow \text{throughput} = \frac{S}{RTT} \frac{3}{4} \frac{1.6}{\sqrt{p}} = \frac{1.2S}{RTT\sqrt{p}}$$

Recap: TCP/Reno Throughput Modeling

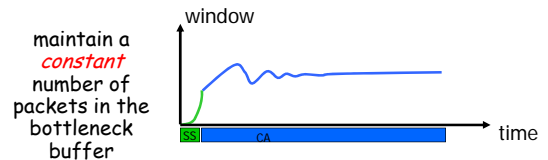
$$\Delta W = \begin{cases} \frac{1}{W} & \text{if the packet is not lost} \\ -\frac{W}{2} & \text{if packet is lost} \end{cases}$$

$$\text{mean of } \Delta W = (1-p) \frac{1}{W} + p(-\frac{W}{2})$$

$$\Rightarrow \text{mean of } W = \sqrt{\frac{2(1-p)}{p}} \approx \frac{1.4}{\sqrt{p}}, \text{ when } p \text{ is small}$$

$$\Rightarrow \text{throughput} \approx \frac{1.4S}{RTT\sqrt{p}}, \text{ when } p \text{ is small}$$

Recap: TCP/Vegas CA algorithm



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for every RTT
{
  if W - W/RTT RTT_min < alpha then W ++
  if W - W/RTT RTT_max > alpha then W --
}
for every loss
  W := W/2
    
```

TCP/Vegas Dynamics

$$\Delta w_{RTT} \approx -(w - xRTT_{\min} - \alpha)$$

$$\Delta w_{\text{unit-time}} = -\left(\frac{w}{RTT} - \frac{x}{RTT} RTT_{\min} - \frac{\alpha}{RTT}\right) = \frac{x}{RTT} RTT_{\min} + \frac{\alpha}{RTT} - x$$

$$\Delta x = \frac{\Delta w_{\text{unit-time}}}{RTT} = \frac{x}{RTT^2} (RTT_{\min} + \frac{\alpha}{x} - RTT)$$

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TCP/Reno vs. TCP/Vegas

	TCP/Reno	TCP/Vegas
Congestion signal	loss rate p	queuing delay T_{queuing}
Dynamics	$\Delta x = \frac{1}{RTT^2} - p \frac{1}{2} x^2$	$\Delta x = \frac{x}{RTT^2} (RTT_{\min} + \frac{\alpha}{x} - RTT)$
Equilibrium	$x_{\text{reno}} = \frac{\alpha_{\text{reno}}}{RTT \sqrt{p}}$	$x_{\text{vegas}} = \frac{\alpha_{\text{vegas}}}{T_{\text{queuing}}}$

Discussion: Why and why not TCP/Vegas?

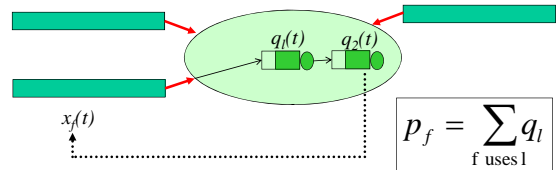
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Interpreting Congestion Measure

- A congestion measure (loss/delay) is a signal from the network to the flows reflecting congestion
- Another way to think of congestion measure is to think of it as "price"
 - price goes up as the rate to a link is getting close to capacity
 - the higher the "price", the lower the rate

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Interpreting Congestion Measure



$$\text{TCP/Reno: } \Delta x = \frac{1}{RTT^2} - p \frac{1}{2} x^2 = \frac{1}{2} x^2 \left(\frac{2}{RTT^2 x^2} - p \right)$$

$$\text{TCP/Vegas: } \Delta x = \frac{x}{RTT^2} (RTT_{\min} + \frac{\alpha}{x} - RTT) = \frac{x}{RTT^2} \left(\frac{\alpha}{x} - T_{\text{queuing}} \right)$$

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Outline

- Recap
- Network bandwidth allocation framework
 - motivation

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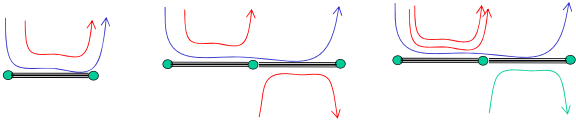
Motivation

- So far our discussion is implicitly on a network with a single bottleneck link; this simplifies design and analysis:
 - efficiency/optimality (high utilization)
 - fully utilize the bandwidth of the link
 - fairness (resource sharing)
 - each flow receives an *equal* share of the link's bandwidth

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Network Resource Allocation

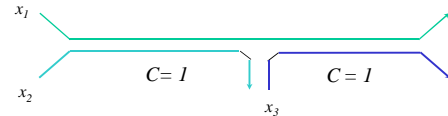
- It is important to understand and design protocols for a general network topology
 - how will TCP allocate resource in a general topology?
 - how should resource be allocated in a general topology?



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Example: TCP/Reno Rates

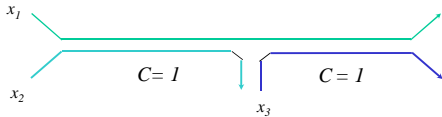
$$\begin{aligned} \text{Rate: } & x_1 = 0.26 \\ & x_2 = x_3 = 0.74 \end{aligned}$$



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Example: TCP/Vegas Rates

$$\begin{aligned} \text{Rates: } & x_1 = 1/3 \\ & x_2 = x_3 = 2/3 \end{aligned}$$

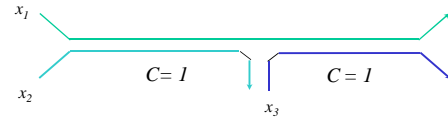


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Example: Maximize Throughput

$$\begin{aligned} \max_{x_f \geq 0} & \quad \sum_f x_f \\ \text{subject to} & \quad x_1 + x_2 \leq 1 \\ & \quad x_1 + x_3 \leq 1 \end{aligned}$$

$$\begin{aligned} \text{Optimal: } & x_1 = 0 \\ & x_2 = x_3 = 1 \end{aligned}$$



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Example: Max-min Fairness

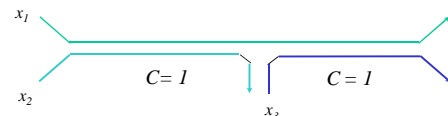


- Max-min fairness: maximizes the throughput of the flow receiving the minimum (of resources)
 - Justification: John Rawls, *A Theory of Justice* (1971)
 - http://en.wikipedia.org/wiki/John_Rawls
 - This is a resource allocation scheme used in ATM and some other network resource allocation proposals

Example: Max-Min

$$\begin{aligned} \max_{x_f \geq 0} & \quad \min \{x_f\} \\ \text{subject to} & \quad x_1 + x_2 \leq 1 \\ & \quad x_1 + x_3 \leq 1 \end{aligned}$$

$$\text{Rates: } x_1 = x_2 = x_3 = 1/2$$



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Network Resource Allocation Using Utility Functions

- A set of flows F
- If x_f is the rate of flow f , then the utility to flow f is $U_f(x_f)$, where $U_f(x_f)$ is a concave utility function.
- Maximize aggregate utility, subject to capacity constraints

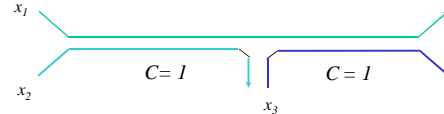
$$\begin{aligned} & \max \sum_{f \in F} U_f(x_f) \\ & \text{subject to } \sum_{f: f \text{ uses link } l} x_f \leq c_l \text{ for any link } l \\ & \text{over } x \geq 0 \end{aligned}$$

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Example: Proportional Fairness

$$\begin{aligned} & \max_{x_f \geq 0} \sum_f \log x_f \quad U_f(x_f) = \log(x_f) \\ & \text{subject to } x_1 + x_2 \leq 1 \\ & \quad \quad \quad x_1 + x_3 \leq 1 \end{aligned}$$

Optimal: $x_1 = 1/3$
 $x_2 = x_3 = 2/3$

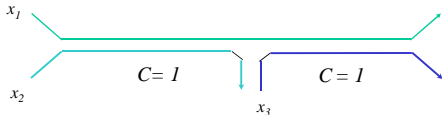


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Example 3: a Utility Function

$$\begin{aligned} & \max_{x_f \geq 0} -\frac{1}{4x_1} - \frac{1}{x_2} - \frac{1}{x_3} \\ & \text{subject to } x_1 + x_2 \leq 1 \\ & \quad \quad \quad x_1 + x_3 \leq 1 \end{aligned} \quad U_f(x_f) = -\frac{1}{RTT^2 x_f}$$

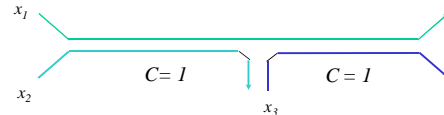
Optimal: $x_1 = 0.26$
 $x_2 = x_3 = 0.74$



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Summary: Allocation

Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
TCP/Vegas	1/3	2/3	2/3
Max Throughput	0	1	1
Max-min	1/2	1/2	1/2
Max sum log(x)	1/3	2/3	2/3
Max sum of -1/(RTT ² x)	0.26	0.74	0.74



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Questions

$$\begin{aligned} & \max \sum_{f \in F} U_f(x_f) \\ & \text{subject to } \sum_{f: f \text{ uses link } l} x_f \leq c_l \text{ for any link } l \\ & \text{over } x \geq 0 \end{aligned}$$

- Forward engineering:** which allocation objective when allocation by optimization?
- Reverse engineering:** what are the objectives of TCP/Reno, TCP/Vegas?

Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
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Max throughput	0	1	1
Max-min	1/2	1/2	1/2
Max sum log(x)	1/3	2/3	2/3
Max sum of -1/(RTT ² x)	0.26	0.74	0.74

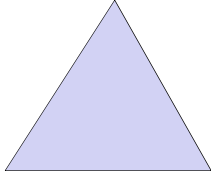
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Outline

- Recap
- Bandwidth allocation framework
 - Motivation
 - Nash Bargaining Solution (NBS)

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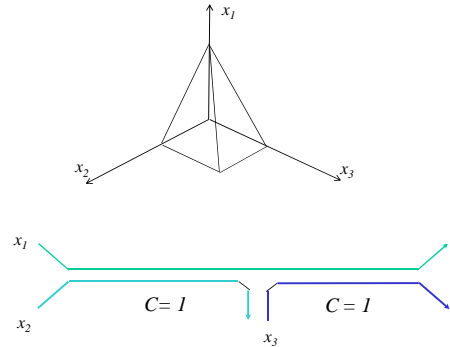
Network Bandwidth Allocation Using Nash Bargain Solution (NBS)



- High level picture
 - given the feasible set of bandwidth allocation, we want to pick an allocation point that is efficient and fair
- The determination of the allocation point should be based on "first principles" (axioms)

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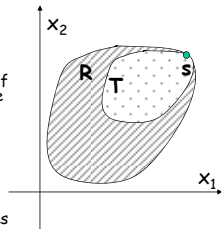
Network Bandwidth Allocation: Feasible Region



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Nash Bargain Solution (NBS)

- Assume a finite, convex feasible set
- Axioms
 - Pareto optimality
 - impossibility of increasing the rate of one user without decreasing the rate of another
 - symmetry
 - a symmetric feasible set yields a symmetric outcome
 - invariance of linear transformation
 - the allocation must be invariant to linear transformations of users' rates
 - independence of irrelevant alternatives
 - assume s is an allocation when feasible set is R , $s \in T \subset R$, then s is also an allocation when the feasible set is T



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Nash Bargain Solution (NBS)

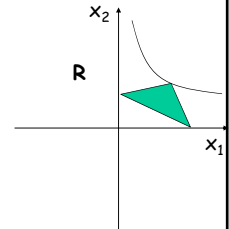
- Surprising result by John Nash (1951)
 - the rate allocation point is the feasible point which maximizes

$$x_1, x_2, \dots, x_F$$

- This is equivalent to maximize

$$\sum_f \log(x_f)$$

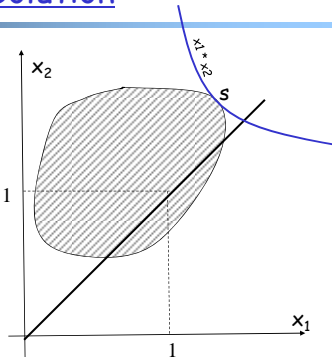
- In other words, assume each flow f has utility function $\log(x_f)$
- I will give a proof for $F = 2$
 - think about $F > 2$



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Nash Bargain Solution

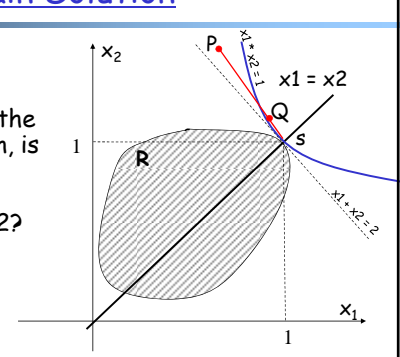
- Assume s is the feasible point which maximizes $x_1 * x_2$
- Scale the feasible set so that s is at $(1, 1)$
- Question: after the transformation, is s still the point maximizing $x_1 * x_2$?



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Nash Bargain Solution

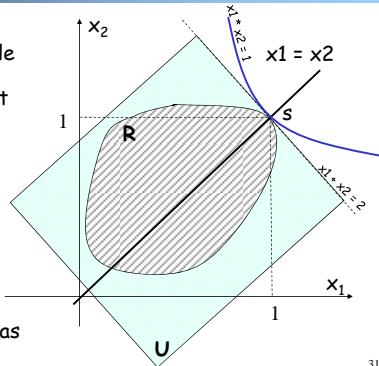
Question: after the transformation, is there any feasible point with $x_1 + x_2 > 2$?



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Nash Bargain Solution

- Consider the symmetric rectangle U containing the original feasible set
- > According to symmetry and Pareto, s is the allocation when feasible set is U
- According to independence of irrelevant alternatives, the allocation of R is s as well.



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NBS \Leftrightarrow Proportional Fairness

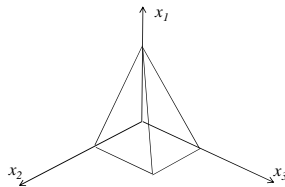
- Allocation is proportionally fair if for any other allocation, aggregate of proportional changes is non-positive, e.g. if x_f is a proportional-fair allocation, and y_f is any other feasible allocation, then require

$$\sum_f \frac{y_f - x_f}{x_f} \leq 0$$

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Summary: Allocation Schemes

- Max throughput
- Max-min
- Proportional fair
 - NBS



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