Wireless PHY:
Modulation and Demodulation

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Outline

- Admin and recap
- Frequency domain examples
- Basic concepts of modulation
- Amplitude modulation
- Amplitude demodulation
  - frequency shifting
Admin

- First assignment to be posted by this weekend

- Any feedback on pace and coverage
Recap: Fourier Series of Periodic Function

- A periodic function \( g(t) \) with period \( T \) on \([a, a+T]\) can be decomposed as:
  \[
  g(t) = \sum_{k=-\infty}^{\infty} G[k] e^{j2\pi \frac{k}{T} t}
  \]
  \[
  G[k] = \frac{1}{T} \int_{a}^{a+T} g(t) e^{-j2\pi \frac{k}{T} t} \, dt
  \]

- For periodic function with period 1 on \([0, 1]\)
  \[
  g(t) = \sum_{k=-\infty}^{\infty} G[k] e^{j2\pi kt}
  \]
  \[
  G[k] = \int_{0}^{1} g(t) e^{-j2\pi kt} \, dt
  \]
Fourier Transform

For those who are curious, we do not need it formally.

Problem: Fourier series for periodic function $g(t)$, what if $g(t)$ is not periodical?

Approach:
- Truncate $g(t)$ beyond $[-L/2, L/2]$ (i.e., set $= 0$) and then repeat to define $g^L(t)$

$$g^L(t) = \sum_{k=-\infty}^{\infty} G^L[k] e^{j2\pi \frac{k}{L} t}$$

$$G^L[k] = \frac{1}{L} \int_{-L/2}^{L/2} g^L(t) e^{-j2\pi \frac{k}{L} t} dt$$
Fourier Transform

\[ G^L[k] = \frac{1}{L} \int_{-L/2}^{L/2} g^L(t) e^{-j2\pi \frac{k}{L} t} \, dt \]

Define \( f_k = \frac{k}{L} \) \( \Delta f = \frac{1}{L} \) \( \hat{G}(f_k) = \int_{-\infty}^{\infty} g^L(t) e^{-j2\pi f_k t} \, dt \)

\[ G^L[k] = \frac{1}{L} \int_{-L/2}^{L/2} g^L(t) e^{-j2\pi \frac{k}{L} t} \, dt \rightarrow G^L[k] = \Delta f \, \hat{G}(f_k) \]

\[ g^L(t) = \sum_{k=-\infty}^{\infty} \hat{G}(f_k) e^{j2\pi f_k t} \Delta f \approx \int \hat{G}(f) e^{j2\pi f t} \, df \]

Let L grow to infinity, we derive Fourier Transform:

\[ \hat{G}(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} \, dt \quad g(t) = \int_{-\infty}^{\infty} \hat{G}(f) e^{j2\pi f t} \, df \]
Fourier Series vs Fourier Transform

- Fourier series
  - For periodical functions, e.g., \([0, 1]\)

\[ g(t) = \sum_{k=-\infty}^{\infty} G[k] e^{j2\pi kt} \]

\[ G[k] = \int_{0}^{1} g(t) e^{-j2\pi kt} \, dt \]

- Fourier transform
  - For non periodical functions

\[ g(t) = \int_{-\infty}^{\infty} \hat{G}(f) e^{j2\pi ft} \, df \]

\[ \hat{G}(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} \, dt \]

Recap: Discrete Domain Analysis

- **FFT**: Transforming a sequence of numbers $x_0$, $x_1$, ..., $x_{N-1}$ to another sequence of numbers $X_0$, $X_1$, ..., $X_{N-1}$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i k}{N} n} \quad k = 0, \ldots, N - 1$$

- **Note** $G[k] = \int_{0}^{1} g(t) e^{-j2\pi k t} \, dt \approx \sum_{n=0}^{N-1} g\left(\frac{n}{N}\right) e^{-j2\pi k \frac{n}{N}} \frac{1}{N}$
Recap: Discrete Domain Analysis

- FFT: Transforming a sequence of numbers $x_0, x_1, \ldots, x_{N-1}$ to another sequence of numbers $X_0, X_1, \ldots, X_{N-1}$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn} \quad k = 0, \ldots, N - 1$$

- Interpretation: consider $x_0, x_1, \ldots, x_{N-1}$ as sampled values of a periodical function defined on $[0, 1]$

$\Rightarrow$ $X_k$ is the coefficient (scaled by $N$) for $k$ Hz harmonics if the FFT $N$ samples span one sec
FFT Analysis vs Sample Rate

\[ X_1 \quad X_2 \quad X_{N_{\text{fft}}/2} \]

\[ N_{\text{fft}} = N_{\text{sample}} \quad 1\text{Hz} \quad 2\text{Hz} \quad N_{\text{fft}}/2 \text{ Hz} \]

\[ \frac{N_{\text{sample}}}{N_{\text{fft}}} \quad \frac{2N_{\text{sample}}}{N_{\text{fft}}} \quad \frac{N_{\text{sample}}}{2} \]

The freq. analysis resolution:

\[ \frac{N_{\text{sample}}}{N_{\text{fft}}} \]
Frequency Domain Analysis Examples Using GNURadio

- spectrum_2sin_plus
  - Audio
  - FFT Sink
  - Scope Sink
  - Noise
Frequency Domain Analysis Examples Using GNURadio

- spectrum_1sin_rawfft
  - Raw FFT
Frequency Domain Analysis Examples Using GNURadio

- `spectrum_2sin_multiply_complex`
  - Multiplication of a sine first by
    - a real sine and then by
    - a complex sine
to observe spectrum
Takeaway from the Example

- Advantages of I/Q representation
I/Q Multiplication Also Called Quadrature Mixing

(a) spectrum of complex signal $x(t)$
(b) spectrum of complex signal $x(t)e^{j2f_0t}$
(c) spectrum of complex signal $x(t)e^{-j2f_0t}$
Basic Question: Why Not Send Digital Signal in Wireless Communications?

- Signals at undesirable frequencies
  - Suppose digital frame repeat every T seconds, then according to Fourier series decomposition, signal decomposes into frequencies at $1/T$, $2/T$, $3/T$, ...
  - Let $T = 1$ ms, generates radio waves at frequencies of 1 KHz, 2 KHz, 3 KHz, ...

![Digital signal diagram](image)
## Frequencies are Assigned and Regulated

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th>USA</th>
<th>Japan</th>
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<tr>
<td><strong>Cordless Phones</strong></td>
<td><strong>CT1+</strong> 885 - 887, 930 - 932 <strong>CT2</strong> 864 - 868 <strong>DECT</strong> 1880 - 1900</td>
<td><strong>PACS</strong> 1850 - 1910, 1930 - 1990 <strong>PACS-UB</strong> 1910 - 1930</td>
<td><strong>PHS</strong> 1895 - 1918 <strong>JCT</strong> 254 - 380</td>
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<td><strong>Wireless LANs</strong></td>
<td><strong>IEEE 802.11</strong> 2400 - 2483 <strong>HIPERLAN 2</strong> 5150 - 5350, 5470 - 5725</td>
<td>902 - 928 <strong>IEEE 802.11</strong> 2400 - 2483 5150 - 5350, 5725 - 5825</td>
<td><strong>IEEE 802.11</strong> 2471 - 2497 5150 - 5250</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td><strong>RF-Control</strong> 27, 128, 418, 433, 868</td>
<td><strong>RF-Control</strong> 315, 915</td>
<td><strong>RF-Control</strong> 426, 868</td>
</tr>
</tbody>
</table>

The maximum number of bits that can be transmitted per second by a physical channel is:

$$W \log_2 (1 + \frac{S}{N})$$

where $W$ is the frequency range of the channel, and $S/N$ is the signal noise ratio, assuming Gaussian noise.
Frequencies for Communications

VLF = Very Low Frequency
LF = Low Frequency
MF = Medium Frequency
HF = High Frequency
VHF = Very High Frequency

UHF = Ultra High Frequency
SHF = Super High Frequency
EHF = Extra High Frequency
UV = Ultraviolet Light

Frequency and wave length:
\[ \lambda = \frac{c}{f} \]

wave length \( \lambda \), speed of light \( c \approx 3 \times 10^8 \text{m/s} \), frequency \( f \)
Why Not Send Digital Signal in Wireless Communications?

At 3 KHz, 
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^3} = 100 km \]

Antenna too large!
Use modulation to transfer to higher frequency
Outline

- Recap
- Frequency domain examples
- Basic concepts of modulation
Basic Concepts of Modulation

- **The information source**
  - Typically a low frequency signal
  - Referred to as baseband signal

- **Carrier**
  - A higher frequency sinusoid
  - Example: \( \cos(2\pi 10000t) \)

- **Modulated signal**
  - Some parameter of the carrier (amplitude, frequency, phase) is varied in accordance with the baseband signal
Types of Modulation

- **Analog modulation**
  - Amplitude modulation (AM)
  - Frequency modulation (FM)
  - Double and signal sideband: DSB, SSB

- **Digital modulation**
  - Amplitude shift keying (ASK)
  - Frequency shift keying: FSK
  - Phase shift keying: BPSK, QPSK, MSK
  - Quadrature amplitude modulation (QAM)
Outline

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- Amplitude modulation
Example: Amplitude Modulation (AM)

- **Block diagram**
  
  \[ x(t) \xrightarrow{m} x \xrightarrow{+} x_{AM}(t) = A_c [1 + mx(t)] \cos w_c t \]

- **Time domain**
  
  Signal information is contained in the sidebands.

- **Frequency domain**
  
  \[ X(f) \xrightarrow{f} X_{AM}(f) \]
Example: am_modulation Example

- Setting
  - Audio source (sample 32K)
  - Signal source (300K, sample 800K)
  - Multiply

- Two Scopes

- FFT Sink
Example AM Frequency Domain

Note: There is always the negative freq. in the freq. domain.
Problem: How to Demodulate AM Signal?

Amplitude Modulation (AM)

- Block Diagram
- Time Domain
- Frequency Domain

\[ x(t) = A_c \cos(c t) \]

\[ x_{AM}(t) = A_c [1 + m x(t)] \cos(c t) \]

Signal information is contained in the sidebands.

\[ X(f) \]

\[ X_{AM}(f) \]
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Design Option 1

- **Step 1:** Multiply signal by $e^{-j2\pi f c t}$
  - Implication: Need to do complex multiple multiplication
Design Option 1 (After Step 1)
Design Option 1 (Step 2)

- Apply a Low Pass Filter to remove the extra frequencies at $-2f_c$
Design Option 1 (Step 1 Analysis)

- How many complex multiplications do we need for Step 1 (Multiply by $e^{-j2\pi fc_t}$)?
Design Option 2: Quadrature Sampling
Quadrature Sampling: Upper Path (cos)
Quadrature Sampling: Upper Path (cos)
Quadrature Sampling: Upper Path (cos)
Quadrature Sampling: Lower Path (sin)
Quadrature Sampling: Lower Path (\(\sin\))
Quadrature Sampling: Lower Path (sin)
Quarature Sampling: Putting Together
Exercise: SpyWork

- Setting: a scanner scans 128KHz blocks of AM radio and saves each block to a file.

- SpyWork: Scan the block in a saved file to find radio stations and tune to each station (each AM station has 10 KHz)