CPSC455b: Solution Set for Hour Exam

Problem 1

(a) Nash Equilibrium.

(b) Dominant-strategy Equilibrium.

(c) Bayes-Nash Equilibrium.

(d) True.

(e) Pareto optimal.

Problem 2

(a) In a *mechanism-design problem*, i, n, A, T, O, $\{u^i\}$, $\{v^i\}$, and $\{p^i\}$ are as in Question 1. It is called a *maximization* mechanism-design problem if the output o is of the form

$$o(t) = arg\max_o g(o,t),$$

where $g(\cdot,\cdot)$ is a real-valued objective function. It is called a *utilitarian* mechanism-design problem if $g(o,t)$ is of the form

$$g(o,t) = \sum_{i=1}^{n} v^i(o,t^i)$$

(b) See definition 7 in Section 3 of the Nisan-Ronen paper "Algorithmic mechanism design" [NR1].

(c) As explained in Sections 3 and 4 of [NR1], MinWork is a VCG mechanism and is therefore truthful.

Problem 3

(a) See pages 2 and 3 of [FPSS].

(b) See pages 5 and 6 of [FPSS].

Problem 4

(a) See Definition 2.7 in [GHKW].

(b) A single-price auction that sells k items to the k highest bidders at the $(k+1)$-st highest price.

(c) See the paragraph directly after Definition 2.7 in [GHKW].

Problem 5

(a) The auctioneer has a set S of items for sale. Each bidder i has a (private) function $v^i()$, the meaning of which is that $v^i(R)$ is the value that i assigns to R, where R is an arbitrary subset of S. If the input is a vector $w = (w^1(), \ldots, w^n())$ of valuations, the mechanism must compute an allocation $A(w)$, which is a vector $(R^1, \ldots, R^n)$ of n disjoint subsets of S, and a vector $p(w) = (p^1, \ldots, p^n)$ of prices. The set R^i consists of those items sold to agent i. The goal of the mechanism is to maximize the sum of valuations, *i.e.*, to maximize $\sum_{i=1}^{n} v^i(A(v), v^i)$, where $v^i(A(v), v^i) \equiv v^i(S^i)$ if $A(v) = (S^1, \ldots, S^n)$.

(b) It is NP-hard to find an allocation that maximizes the sum of the valuations.

(c) A "VCG mechanism based on $k()$" is a mechanism whose allocation function is $k()$ and, when the agents' input is a valuation vector $w = (w^1, w^2, \ldots, w^n)$, assigns to agent i the payment

$$p^i = \left(\sum_{j \neq i} w^j(k(w), w^j) \right) + h^{-i}(w^{-i}).$$

(d) First, it is important to be precise about the meaning of the phrase "improve the overall allocation." Nisan and Ronen mean that agent i can lie to improve the overall allocation by giving w^i as input instead of its true valuation v^i if $v = (v^i, w^{-i})$ and

$w = (w^i, w^{-i})$ are such that

$$v^i(k(w), v^i) + \left(\sum_{j \neq i} w^j(k(w), w^j) \right) > v^i(k(v), v^i) + \left(\sum_{j \neq i} w^j(k(v), w^j) \right).$$

(The paper does not say this in so many words, but it is the only meaning that is consistent with everything else that is said.)

The utility of agent i is $u^i = v^i + p^i$. Plug in the payment defined in part (c) above. Agent i's utility on input vector w is

$$v^i(k(w), v^i) + \left(\sum_{j \neq i} w^j(k(w), w^j) \right) + h^{-i}(w^{-i}),$$

and its utility on input vector v is

$$v^i(k(v), v^i) + \left(\sum_{j \neq i} w^j(k(v), w^j) \right) + h^{-i}(v^{-i}).$$

The motivation for agent i to lie is to increase its utility. Because $w^{-i} = v^{-i}$, its utility is increased when it inputs w^i instead of v^i if and only if the allocation is improved.