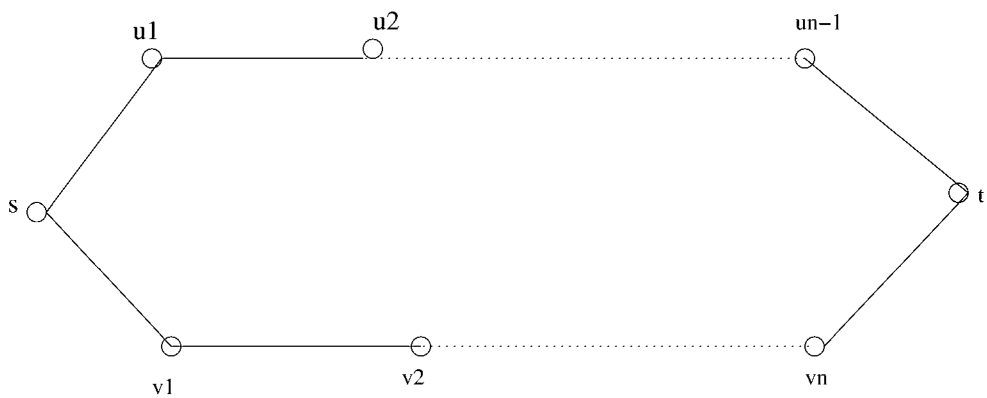
CPSC455b: Solution Set for Written Homework Assignment #2

Problem 1

(a) Here, the mechanism computes the shortest path $s - - - t$.



In the above graph of $2n+1$ vertices, let the real cost of every edge be 1. Consider $S = \{(s,u_1),(s,v_1)\}$. For $x > 1$, if $a^{(s,u_1)} = a^{(s,v_1)} = x$, then (s,u_1) can get strictly higher utility, while (s,v_1) loses nothing.

$$u^{(s,u_1)}(o', t^{(s,u_1)}) = x \ > \ 1 = u^{(s,u_1)}(o, t^{(s,u_1)})$$

$$u^{(s,v_1)}(o', t^{(s,v_1)}) = \ 0 \ = u^{(s,v_1)}(o, t^{(s,v_1)})$$

(b) This definition does not necessarily capture one's intuition, because it does not take the following situation into account: One member of a strategizing group might benefit greatly from (the group's) lie, and another member might lose slightly; then the member who benefitted could compensate the one who lost. Intuitively, this could also be considered a successful group strategy, but it is not considered successful according to the definition given.

Problem 2

(a) If $a_e = 0$, then Lemma 4.1 of the Roughgarden-Tardos paper [RT] implies that f is a Nash flow $\Leftrightarrow$ for any i, P, P', with $f_P > 0$, $\sum_{e \in P} b_e \leq \sum_{e \in P'} b_e \Leftrightarrow f$ is an optimal flow.

If $b_e = 0$, then Lemma 4.1 of [RT] implies that f is a Nash flow $\Leftrightarrow$ for any i, P, P', with $f_P > 0$, $\sum_{e \in P} a_e f_e \leq \sum_{e \in P'} a_e f_e \Leftrightarrow$ for any i, P, P', with $f_P > 0$, $\sum_{e \in P} 2a_e f_e \leq \sum_{e \in P'} 2a_e f_e \Leftrightarrow f$ is an optimal flow. (This is Corollary 4.2 of [RT].)

(b) See the last paragraph on page 21 of the Roughgarden-Tardos paper.

Problem 3

(a) For all a^{-i}, a feasibly dominant action a^i guarantees that agent i will achieve the highest utility it can achieve using any strategy that it can find in polynomial time, *given* the limitations imposed by its knowledge function b^i. For all a^{-i}, a dominant action a^i guarantees that agent i will achieve the highest utility it can achieve using any strategy (unconditionally). Clearly if a^i satisfies the definition of a dominant strategy, then it satisfies the definition of a feasibly dominant strategy for any knowledge function b^i.

The question about the form of b^i for an agent i that has a dominant strategy was a proverbial "trick question." If i has a dominant strategy, then it will just play that strategy without ever evaluating a knowledge function; thus, it does not matter what form b^i has.

(b) We first show that it is CMAP. Let v_j^i be the negation of the cost of agent i's jth edge. Let O be the set of bit vectors corresponding to desired trees. Obviously, the cost of an edge can be unboundedly large, *i.e.*, the type can be unboundedly small. The valuation of an agent depends only on her own edges in the output tree. If we increase the costs of the edges an agent owns, (*i.e.*, decrease its type), its evaluation of every output will strictly decrease. For the forcing condition, note that the tree corresponding to output y must contain an edge that is not in the tree corresponding to output x. Let the type of this edge be α. As α goes to $-\infty$, the cost of the edge goes to $+\infty$. The other edges in the tree corresponding to y have fixed costs, and thus the overall valuation of y goes to $-\infty$.

To see that it is an "NP-complete" mechanism-design problem, note that the tree to be computed is an optimal Steiner tree. It is well-known that finding an optimal Steiner tree is NP-hard.

Problem 4

Let v^i, w^i, m', $\overline{R}5$, $\overline{R'}$, $\overline{R'_1}$, and $\overline{R'_2}$ be as in the paper. Let ϕ_k be the distribution ϕ conditioned on $(w^{k+1}, \ldots, w^n)$, and let p^1 be the price for each item that maximizes revenue from the k highest bidders according to ϕ_k.

Obviously, the auction satisfies IR. To see that it satisfies IC, fix the declaration of other agents and consider agent i. There are three cases:

- $v^i \geq w^{k+1}$ and $v^i \geq p^1$. Then agent i wants to win and will do so if she tells the truth.
- $w^{k+1} \leq v^i < p^1$. Then the only way that agent i could affect her profit by lying is to declare $w^i > p^1$, but this would make her lose money.
- $v^i < w^{k+1}$. If the agent lied to win the auction, she would pay at least w^{k+1} and thus lose money.

Next, we show that $2\overline{R} \geq \overline{R'} = \overline{R'_1} + \overline{R'_2}$, where $\overline{R'_1}$ (*resp.*, $\overline{R'_2}$) is the expected revenue when m' picks (*resp.*, does not pick) the k highest bids:

- m' picks the k highest bids. The case in which $k = n$ is trivial. So assume that $n > k$, and fix $(v^{k+1}, \ldots, v^n)$. Because p^1 is optimal given $(v^{k+1}, \ldots, v^n)$, the expected revenue in this case cannnot be greater than kp^1. Integrating over all possible $(v^{k+1}, \ldots, v^n)$, we get $\overline{R} \geq \overline{R'_1}$.
- m' does not pick the k highest bids. Fix $(v^{k+1}, \ldots, v^n)$. Note $p^1 \geq v^{k+1}$, because m can guarantee a revenue of kv^{k+1}. However, the payment in m' must be no more than v^{k+1}, because otherwise IR would not be satisfied. Therefore $\overline{R} \geq \overline{R'_2}$.

Combining the above results gives $2\overline{R} \geq \overline{R'} = \overline{R'_1} + \overline{R'_2}$.

Problem 5

Consider bid vectors with n_h bids of value $h > 1$, n_1 bids of value 1, and $\frac{n}{2}$ bids at *different* values each less than $\frac{1}{n^2}$ (*e.g.*, $\frac{1}{n^3}$, $\frac{2}{n^3}$, ..., $\frac{n/2}{n^3}$). It is easy to verify that the argument in Goldberg *et al.*'s proof of Theorem 5.1 still holds in this case. Therefore, we can have $\Omega(n)$ different values for sufficiently large n.

Note that the above construction is valid only for sufficiently large n and is not exactly optimal. More generally, the proof works for up to $n - \delta(h)$ different values, where $\delta(h)$ is a function of h. (Deriving an exact formula for $\delta(h)$ is beyond the scope of this homework problem.)