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Economics and Computation

Computer Science 455/555 and Economics 425/563
Problem Set 3: Combinatorial Auctions and Sponsored Search Auctions
10/23/08

This problem set is due on **Tuesday, 11/04/08**. The first three question were originally part of Problem Set 2. They are due jointly with Problem Set 3 and are recorded for completeness.

1. Exercise 11.3 in (Nisan, Roughgarden, Tardos, and Vazirani 2008): Demand Query and Information.

2. Exercise 11.7 in (Nisan, Roughgarden, Tardos, and Vazirani 2008): Ascending auctions with superadditive valuations.

3. Exercise 11.8 in (Nisan, Roughgarden, Tardos, and Vazirani 2008): Ascending auctions and informational efficiency.

4. Exercise 28.2 in (Nisan, Roughgarden, Tardos, and Vazirani 2008): Efficient allocation with separable click-through rate: $\alpha_{ij} = \mu_i \cdot \beta_j$.

5. Exercise 28.4 in (Nisan, Roughgarden, Tardos, and Vazirani 2008): Generalized second price auction with separable click-through rate: $\alpha_{ij} = \mu_i \cdot \beta_j$.

6. Ascending Auction. So far, we have analyzed the sponsored search auction as one of complete information. Let us now consider an ascending auction for the k positions with incomplete (private) information. There are $j = 1, ..., n$ bidders, each with a private valuation $v_j > 0$ and the click-through rate is bidder independent with $\mu_i > 0$ for each position $i = 1, ...,$ with $\mu_1 > \cdots > \mu_k$ and $k < n$.
   The rule of the ascending auction (often referred for the single item auction as Japanese button auction) is as follows. There is a single price which is interpreted as bid for a slot and the price is continuously rising. Each bidder initially presses a button and as soon as he releases the button, he indicates that he has reached his maximal willingness to pay. The price at which j releases the button is the exit price p_j of bidder j. Once a bidder releases the button, he cannot participate in the auction again (i.e. start pressing the button again). An assignment

is made whenever the number of participating bidders drops below the number of unassigned slots. It starts after some bidders have already exited and with only k bidders left, another bidder leaves the auction. Then the k highest bidder (the one who just exited the auction) gets the k-th slot and pays the exit price of bidder $k+1$, or p_{k+1}. More generally, if there are only l bidders left and now bidder l leaves at a price p_l, then he gets the $l-$highest slot and pays (always in the event of a click-through rate) the exit price p_{l+1}.

1. Compute recursively the bidding strategy of bidder i, that is the determination of his exit price as a function of his own valuation and the exit price of all those bidders who have already left the auction before him. Thus formally the bidding strategy of bidder i is as sequence of mappings

   $$b_i^m : \mathbb{R}_+ \times \mathbb{R}_+^m,$$

   where m is the number of bidders who have left already. (Hint: (i) Suppose that until the number of bidders left in the game is less than k, the total number of positions, then

   $$b_i^m = v_i \cdot \mu_k$$

   for all $m < n - k$. Afterwards (ii), with l less than k bidders left, think about what the l bidders are competing for effectively, and think about their maximal willingness to pay for the object over which they effectively compete.
2. Compare the resulting prices to the Vickrey-Clarke-Groves prices which you had computed earlier (in class).

Readings. (Nisan, Roughgarden, Tardos, and Vazirani 2008), Chapter 28. For question 6 if you need additional help, you might find the article by Hal Varian, (Varian 2007) (on google scholar) useful. In the past two lectures, I have assumed some familiarity with linear algebra as we stated the duality results and the linear programming problems. The chapter 5 in (Chvatal 1980) and the chapter 4 in (Vohra 2005) are excellent introductions (and refreshers). An interesting survey article, easy to read, on the economics of internet search, is (Varian 2006). An introduction into the search algorithm used by Google and others is given by (Langville and Meyer 2006).

References

Chvatal, V. (1980): *Linear Programming*. Freeman and Company, New York.

Langville, A., and C. Meyer (2006): *Google's PageRank and Beyond: The Science of Search Engine Rankings*. Princeton University Press, Princeton.

Nisan, N., T. Roughgarden, E. Tardos, and V. Vazirani (2008): *Algorithmic Game Theory*. Cambridge University Press, Cambridge.

Varian, H. (2006): “The Economics of Internet Search,” *Rivisista di Economia Politica*, pp. 177–191.

Varian, H. (2007): “Position Auctions,” *International Journal of Industrial Organization*, 25, 1163–1178.

Vohra, R. (2005): *Advanced Mathematical Economics*. Routledge, Oxford.