CPSC 455/555 // ECON 425/563, Fall 2011, HW5

Undergraduates may pick any 5 of these 6 problems to solve; grad students must do all 6. If an undergraduate chooses to solve all 6, the problems will all be graded and the top 5 scores will be chosen for the overall score.

As in HWs 1 through 4, each Question is worth a total of 10 points.

For Questions 1 and 2, you will need to read Section 26.4 of Nisan *et al.* carefully.

Question 1: Problem 26.1. Note that there is a typo: It's Section 26.4, not 26.5.

Question 2: Problem 26.3.

Question 3:
Consider the boolean functions $f(x_1, \ldots, x_n) = x_1 \wedge \cdots \wedge x_n$ (logical AND), $g(x_1, \ldots, x_n) = \neg f(x_1, \ldots, x_n)$ (logical NAND), and $h(x_1, \ldots, x_n) = \neg(x_1 \vee \cdots \vee x_n)$ (logical NOR).

(a) (5 points) Which of f, g, and h is computable using the simplified Shapley-Shubik computation model of Section 26.5 that was covered in class in Lectures 21 and 22; briefly justify your answers.

(b) (5 points) For each function in part (a) that is computable in this model, show how the price-formation process reveals the value of the function on the input vector (0, 1, 1, 0, 1) and the common prior in which each x_i is chosen independently according to $P(x_i = 0) = 1/4$ and $P(x_i = 1) = 3/4$. That is, give the bids of each agent in each round and the clearing price in each round, give the equilibrium price, and explain why the process terminates when it does. For each function in part (a) that is not computable in this model, construct a common prior on which the process is not guaranteed to converge to the value of the function.

Question 4:
Consider the following variation on the peer-prediction example developed in Section 27.4. There are two types H and L, with $P(H) = P(L) = 0.5$, and two signals h and l, with conditional probabilities $P(h \mid H) = 0.6$ and $P(l \mid L) = 0.8$.

(a) (3 points) Adopting the viewpoint of agent 1, determine the conditional probabilities $P(S^2 = h \mid S^1 = l)$ and $P(S^2 = h \mid S^1 = h)$, where S^1 and S^2 are the random variables representing the signals of agents 1 and 2.

(b) (3 points) Suppose that agent 1 gets signal h. Assuming agent 2 is truthful, verify that the expected score of agent 1 is maximized by truthful reporting. Assume that the *quadratic scoring rule* of Problem 27.7 is used.

(c) (4 points) How could agents 1 and 2 profitably collude?

Question 5:
Provide explicit proofs of the following statements, which are presented without explanation in Chapter 27 in the proofs of Theorems 27.8 and 27.9:

(a) (4 points) [In Theorem 27.8]: All sybils $v \in V$ must be on the same side of the cut as v and thus on the other side of the cut from the source s.

(b) (3 points) [In Theorem 27.9]: Sybils cannot decrease the length of the shortest path.

(c) (3 points) [In Theorem 27.9]: Node v can only affect the value of node w if v is on the shortest path from s to w.

Question 6:
Recall the *centipede game* defined in
`http://www.seas.harvard.edu/courses/cs186/doc/2-game-theory.pdf`. (See Lecture 15.) As explained in the Osborne and Rubinstein book referred to in these notes,

> Two players are involved in a process that they alternatively have the opportunity to stop. Each prefers the outcome when he stops the process in any period t to that in which the other player does so in period $t+1$. However, better still is any outcome that can result if the process is not stopped in either of these periods. After T periods, where T is even, the process ends. ...
>
> Formally, the set of histories in the game consists of all sequences $C(t) = (C, C, \ldots, C)$ of length t, for $0 \leq t \leq T$, and all sequences $S(t) = (C, C, \ldots, C, S)$ consisting of $t-1$ repetitions of C followed by a single S, for $1 \leq t \leq T$. The player function P, *i.e.*, the function of t that tells us which player plays in period t, is defined by $P(C(t)) = 1$ if t is even and $t \leq T-2$, and $P(C(t)) = 2$ if t is odd. Player $P(C(t))$ prefers $S(t+2)$ to $S(t)$ [and prefers $S(t)$] to $S(t+1)$, for $t \leq T-2$; player 1 prefers $C(T)$ to $S(T-1)$ [and prefers $S(T-1)$] to $S(T)$; and player 2 prefers $S(T)$ to $C(T)$.

(a) (3 points) Prove that this game has a unique subgame-perfect NE in which each player chooses S in each period.

(b) (7 points) For any $\epsilon > 0$, define an ϵ-*equilibrium* as a profile of actions with the property that no player has an alternative action that increases his payoff by more than ϵ. Show that, for any positive integer k and any $\epsilon > 0$, there is a T large enough so that the modification of the centipede game in which all payoffs are divided by T has an ϵ-*equilibrium* in which the first player to stop the game does so in period k.