Internet Advertising and the Generalized Second Price Auction:

Selling Billions of Dollars Worth of Keywords

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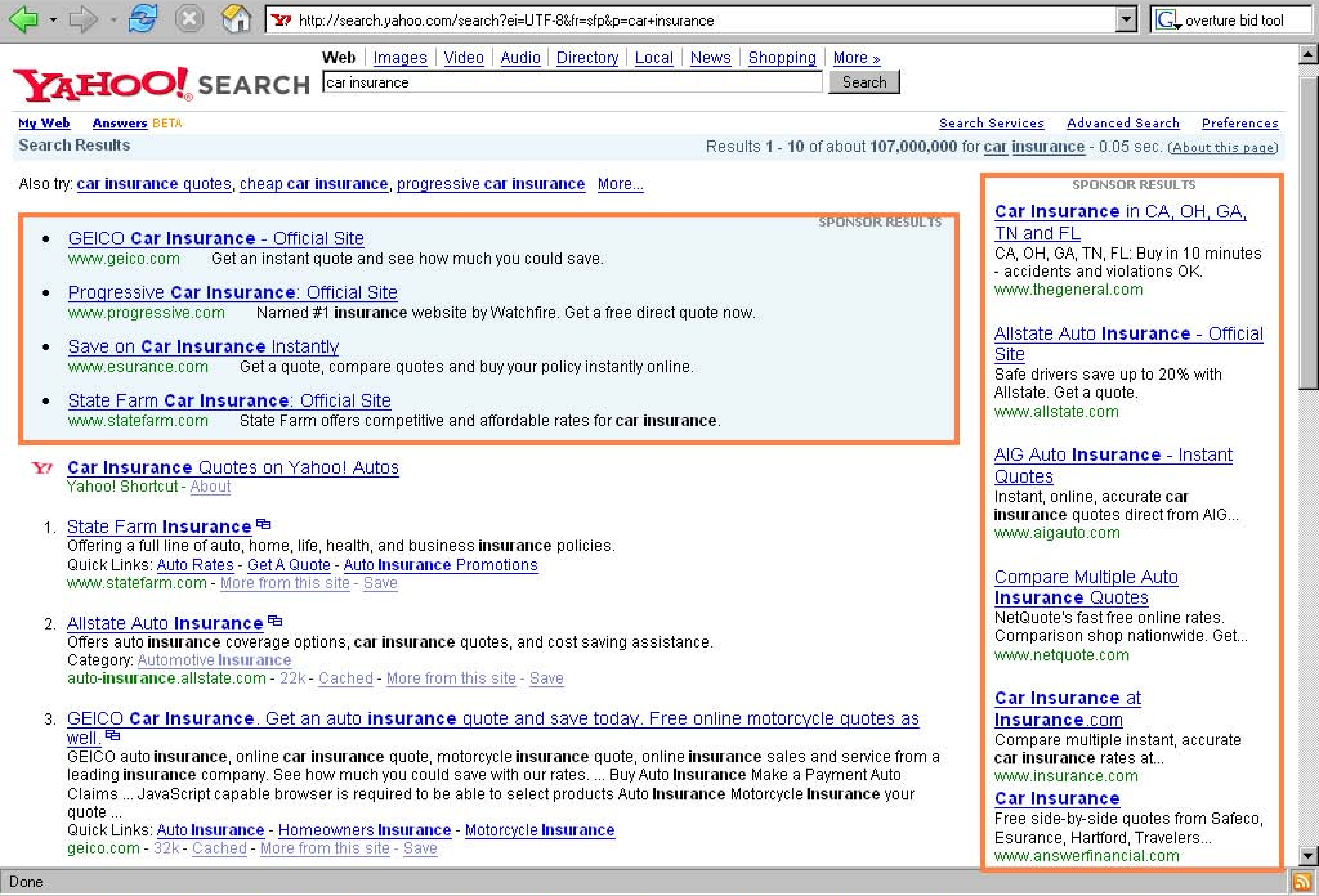
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Type in a search term and we'll show you the Max Bids and listings for that term.

car insurance

Search Cancel

1. **GEICO Car Insurance - Official Site**
   Get an instant quote and see how much you could save.
   www.geico.com
   (Advertiser's Max Bid: $10.49)

2. **Progressive Car Insurance: Official Site**
   Named #1 insurance website by Watchfire. Get a free direct quote now.
   www.progressive.com
   (Advertiser's Max Bid: $10.48)

3. **Save on Car Insurance Instantly**
   Get a quote, compare quotes and buy your policy instantly online. Print your insurance cards in minutes. Quote. Buy. Print. It's that easy to save on car insurance with Esurance.
   www.esurance.com
   (Advertiser's Max Bid: $9.15)

4. **State Farm Car Insurance: Official Site**
   State Farm offers competitive and affordable rates for car insurance. Deep discounts for safe drivers, good students and safety features in your car.
   www.statefarm.com
   (Advertiser's Max Bid: $9.14)

5. **Car Insurance in CA, OH, GA, TN and FL**
   CA, OH, GA, TN, FL: Buy in 10 minutes - accidents and violations OK.
   www.thegeneral.com
   (Advertiser's Max Bid: $8.99)

6. **Allstate Auto Insurance - Official Site**
   Safe drivers save up to 20% with Allstate. Get a quote.
   www.allstate.com
   (Advertiser's Max Bid: $8.50)

Done

A Few Facts about GSP

- Tailored to its environment

- Google's revenue in 2005 \$6.14 B, over 98% from GSP

- Yahoo!'s revenue in 2005 \$5.26 B, over 50% from GSP

- Other companies using GSP and its variations:
  - MSN search
  - Ask.com
  - Many smaller search engines.

History

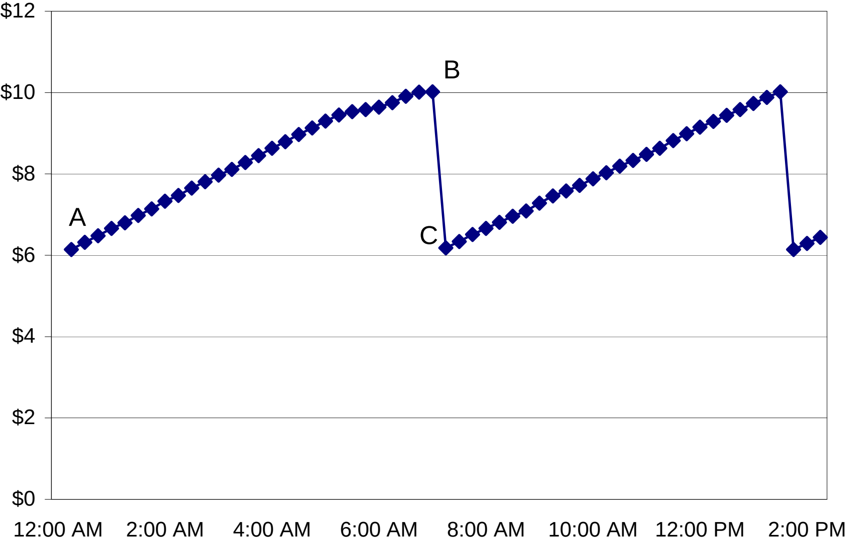
Unlike spectrum auctions and electricity auctions, which were designed essentially from scratch, sponsored search auctions evolved over time.

- Early Internet advertising (1994): per-impression pricing, person-to-person negotiations, no keyword targeting.

- Overture's (1997) generalized first-price auctions:
  - pay-per-click, for a particular keyword
  - completely automated, bids can be changed at any time
  - links are arranged in the descending order of bids
  - pay <u>your own</u> bid

Problem. Generalized First-Price Auction is unstable, because it generally does not have a pure strategy equilibrium, and bids can be adjusted dynamically.

Example. Two slots and three bidders. First slot gets 100 clicks per hour, second slot gets 70. Bidders 1, 2, and 3 have values per click of \$10, \$8, and \$5, respectively. There is no pure strategy equilibrium in the one-shot version of the game. If bidders best respond to each other, they will want to revise their bids as often as possible.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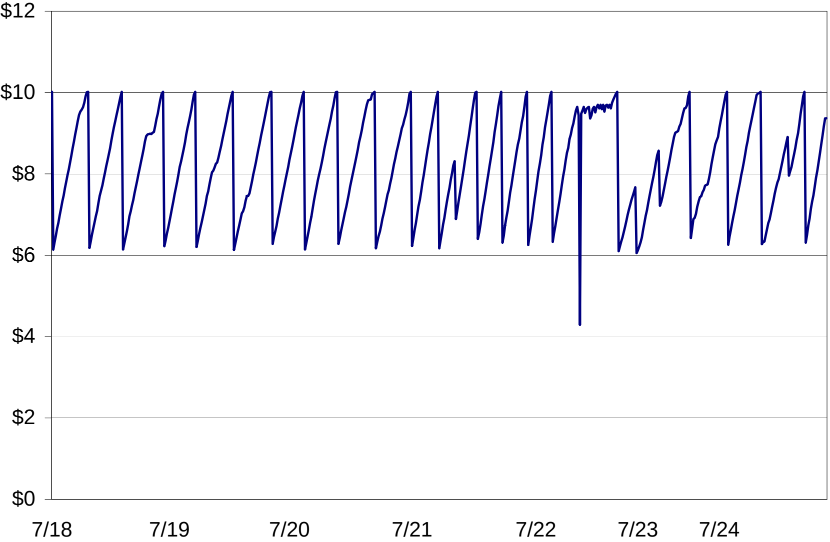

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7/24

History (continued)

- Google's (2002) generalized second-price auction (GSP):
  - pay the bid of the next highest bidder

- Later adopted by Yahoo!/Overture and others.

Generalized Second-Price and Vickrey Auctions

> *"[Google's] unique auction model uses Nobel Prize-winning economic theory to eliminate [. . .] that feeling that you've paid too much."*
>
> — marketing materials at google.com

- With only one slot, GSP is identical to the standard second price auction (a.k.a. Vickrey, VCG).

- With multiple slots, the mechanisms are different
  - GSP charges bidder k the *bid* of bidder $k+1$
  - VCG charges bidder k for his *externality*

Example. Two slots, three bidders. First slot gets 100 clicks per hour, second slot gets 70. Bidders 1, 2, and 3 have values per click of \$10, \$8, and \$5, respectively. If all advertisers bid truthfully, then bids are \$10, \$8, \$5.

Under GSP, payments for slots one and two are \$8 and \$5 per click. Total payments of bidders one and two are \$800 and \$350, respectively.

Under VCG, the second bidder's payment is still \$350. However, the payment of the first advertiser is now \$590: \$350 for the externality that he imposes on bidder 3 (by forcing him out of position 2) and \$240 for the externality that he imposes on bidder 2 (by moving him from position 1 to position 2 and thus causing him to lose $(100 - 70) = 30$ clicks per hour).

Truth-telling is not a dominant strategy under GSP

Per click values are \$10, \$8, and \$5

CTR's are 100 and 70

If everyone bids truthfully, bidder 1's payoff is

$(\$10 - \$8) * 100 = \$200.$

If instead bidder 1 bids \$6, his payoff is

$(\$10 - \$5) * 70 = \$350 > \$200.$

GSP and the Generalized English Auction

$N \geq 2$ slots and $K = N + 1$ advertisers

α_i is the expected number of clicks in position i

s_k is the value per click to bidder k

A clock shows the current price; continuously increases over time

A bid is the price at the time of dropping out

Payments are computed according to GSP rules

Bidders' values are private information, drawn randomly from commonly known distributions

Strategy can be represented by $p_k(i, h, s_k)$

s_k is the value per click of bidder k,

p_k is the price at which he drops out,

i is the number of bidders remaining (including bidder k), and

$h = (b_{i+1}, \ldots, b_{N+1})$ is the history of prices at which bidders $N + 1$, N, ..., $i + 1$ have dropped out.

If bidder k drops out after history h, he pays b_{i+1} (unless the history is empty, then set $b_{i+1} \equiv 0$).

Theorem. *In the unique perfect Bayesian equilibrium of the generalized English auction with strategies continuous in* s_k*, an advertiser with value* s_k *drops out at price*

$$p_k(i, h, s_k) = s_k - \frac{\alpha_i}{\alpha_{i-1}}(s_k - b_{i+1}).$$

In this equilibrium, each advertiser's resulting position and payoff are the same as in the dominant-strategy equilibrium of the game induced by VCG. This equilibrium is ex post: the strategy of each bidder is a best response to other bidders' strategies regardless of their realized values.

1. Payments coincide with VCG

By induction, from the bottom. First,

$$b_{N+1} = s_{N+1},$$

so the payment of bidder N is $\alpha_N s_{N+1}$.

Next,

$$b_N = s_N - \frac{\alpha_N}{\alpha_{N-1}}(s_N - b_{N+1}),$$

so the total payment of bidder $(N-1)$ is

$$\alpha_{N-1} b_N = \alpha_{N-1} s_N - \alpha_N s_N + \alpha_N b_{N+1}$$

$$= s_N(\alpha_{N-1} - \alpha_N) + \alpha_N s_{N+1}.$$

Repeat for b_{N-1}, b_{N-2}, etc. . .

2. The profile is an ex-post equilibrium

By construction, each bidder i is indifferent between his position at b_{i+1} per click and position $i-1$ at b_i per click:

$$\begin{aligned} b_i &= s_i - \frac{\alpha_i}{\alpha_{i-1}}(s_i - b_{i+1}) \\ &\Updownarrow \\ \alpha_{i-1}(s_i - b_i) &= \alpha_i(s_i - b_{i+1}). \end{aligned}$$

Since $s_{i-1} \geq s_i$, this implies that bidder $i-1$ prefers his position $(i-1)$ at b_i per click to position i at b_{i+1} per click, which in turn he prefers to position $i+1$ at b_{i+2} per click, etc. Hence, no bidder wants to reduce his bid.

Similarly, each bidder i prefers his position at b_{i+1} to any position $k < i$ at b_{k+1}, and the price he would have to pay for position k if he wanted to switch there is even greater: b_k. Hence, no bidder wants to increase his bid.

3. Uniqueness (intuition)

By construction, player k is indifferent between getting position i at price b_{i+1} and position $i-1$ at price $p_k = s_k - \frac{\alpha_i}{\alpha_{i-1}}(s_k - b_{i+1})$. Hence, with i players remaining and the next highest bid equal to b_{i+1}, it is a weakly dominated strategy for player k to drop out before the price on the clock reaches $p_k(i, h, s_k)$—the level at which he is indifferent between getting position i and paying b_{i+1} per click and getting position $i-1$ and paying p per click.

Next, if for some set of types it is not optimal to drop out at this "borderline" price level, consider the lowest such type. Once the clock reaches this price level, a player of this type will know that he has the lowest per-click value of the remaining players. But then he will also know that the other remaining players will only drop out at price levels at which he will find it unprofitable to compete with them for the higher positions.

■

Static GSP and Locally Envy-Free Equilibria

Let us now step back from the specific convergence model of the Generalized English Auction and ask a different question. Suppose in the dynamic market, after some initial period, bids stabilize at some values. What can these values be?

Restrictions suggested by the dynamic nature

1. All bidders play static best response

2. *Locally envy-free equilibrium:* No bidder wants to swap positions and payments with a bidder right above him

Varian (2006) imposes the same restrictions ("Symmetric Nash equilibrium").

Matching Advertisers to Positions

Shapley and Shubik (1972): matching with payments

$\alpha_i s_k$ is the value of position–advertiser pair (i, k)

p_{ki} is the payment of advertiser k "to position i"

Advertiser's payoff: $\alpha_i s_k - p_{ki}$

Lemma. *The outcome of any locally envy-free equilibrium of auction GSP is a stable assignment.*

Lemma. *If the number of bidders is greater than the number of available positions, then any stable assignment is an outcome of a locally envy-free equilibrium of GSP.*

“Special” Locally Envy-Free equilibrium:

Strategy profile B^*: $b_i^* = \frac{p^{V,(i-1)}}{\alpha_{i-1}}$ for $i \neq 1$, $b_1^* = s_1$, where

$p^{V,(j)} = (\alpha_j - \alpha_{j+1})s_{j+1} + p^{V,(j+1)}$ (payment of j under VCG)

Theorem. *B^* is a locally envy-free equilibrium of GSP. In this equilibrium, each bidder's position and payment is equal to those in the dominant-strategy equilibrium of VCG. In any other locally envy-free equilibrium, the payments of bidders and the revenue of the seller are at least as high as in B^*.*

Intuition. Bidder j is indifferent between staying in his position and “trading assignments” with a bidder right above him.

$$\alpha_{j-1}(s_j - \frac{p^{V,(j-1)}}{\alpha_{j-1}}) = \alpha_{j-1}s_j - (\alpha_{j-1} - \alpha_j)s_j - p^{V,(j)} = \alpha_j(s_j - \frac{p^{V,(j)}}{\alpha_j}).$$

Side Remark

We assumed that all advertisers were identical along dimensions other than per-click value (e.g., had identical click-through rates). The analyis remains largely the same if instead we assume that the CTRs of different advertisers are multiples of one another, i.e., if any advertiser k assigned to any position i receives $\alpha_i \beta_k$ clicks, where α_i is a position-specific factor and β_k is an advertiser-specific factor. It also generalizes easily to the version of the auction implemented by Google, where bids are multiplied by advertiser-specific "quality scores" γ_k for ranking and pricing purposes.

Conclusions

- GSP looks similar to VCG, but is not the same: GSP is not dominant strategy solvable, and truth-telling is generally not an equilibrium;

- The corresponding Generalized English Auction:
  - has a unique equilibrium and explicit analytic formulas for bid functions, which is very useful for empirical analysis;
  - is a robust mechanism—the equilibrium does not depend on distributions of types, beliefs, etc.

- No other mechanisms encountered in practice are not dominant strategy solvable yet robust.