On the Lindell-Pinkas Secure Computation of Logarithms: From Theory to Practice

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Overview

Introduction

The Lindell-Pinkas $\ln x$ protocol

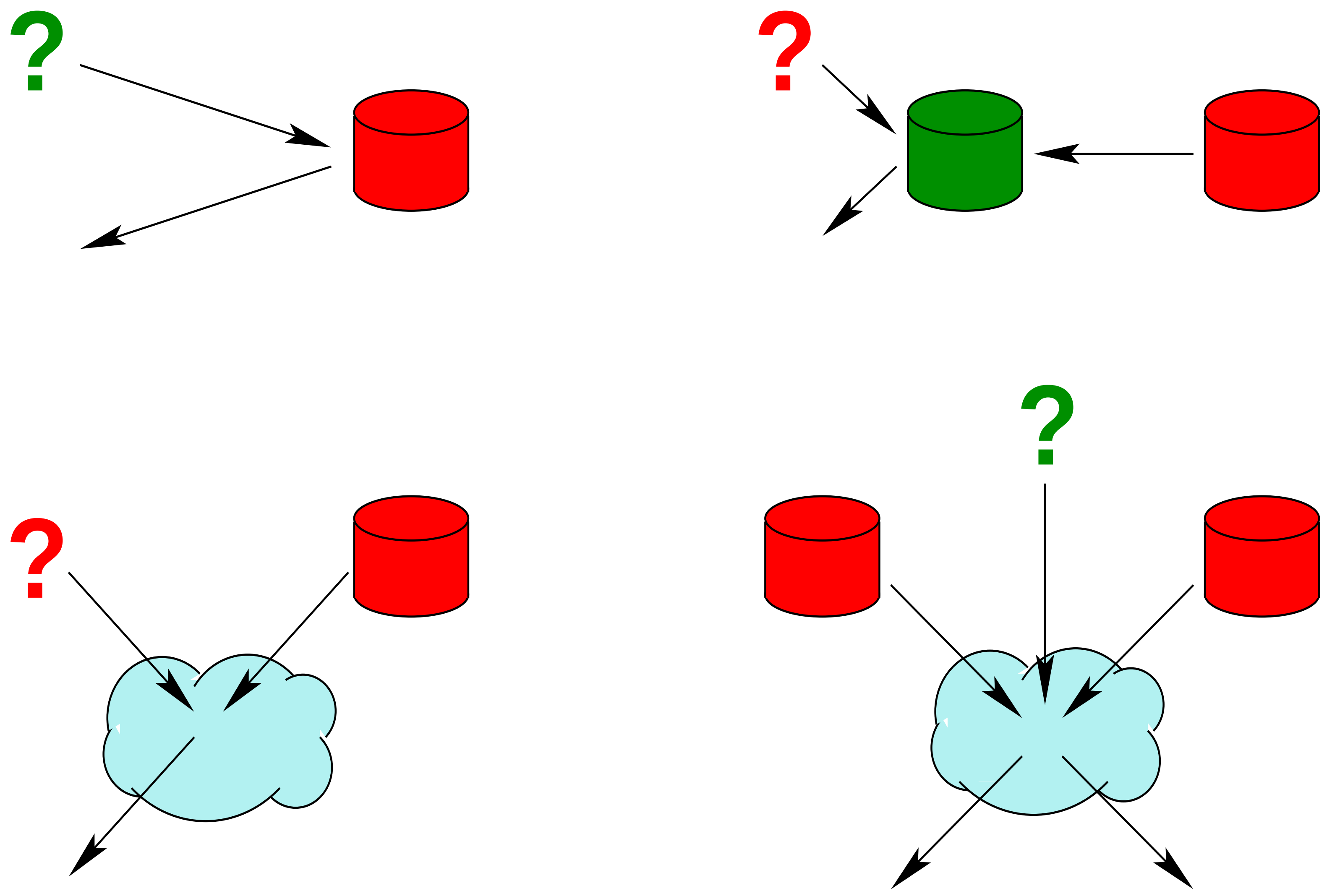
The division problem

Secure non-integer scaling of shared values

Implementation and performance

Conclusion

A variety of PPDM settings



SMC and PPDM

- PPDM dilemmas:
  - **what data to expose** for analysis;
  - **what analyses to allow**.
- Secure multiparty computation – SMC – theoretically eliminates the former, reducing PPDM to the latter.
- **Generic approaches** to achieving SMC are computationally expensive for non-trivial algorithms and large amounts of input data, making them **impractical for PPDM**.
- Lindell, Pinkas, 2000: A **modular, hybrid** SMC approach, combining building blocks implemented through generic or specialized technologies, can be **practical for PPDM**!
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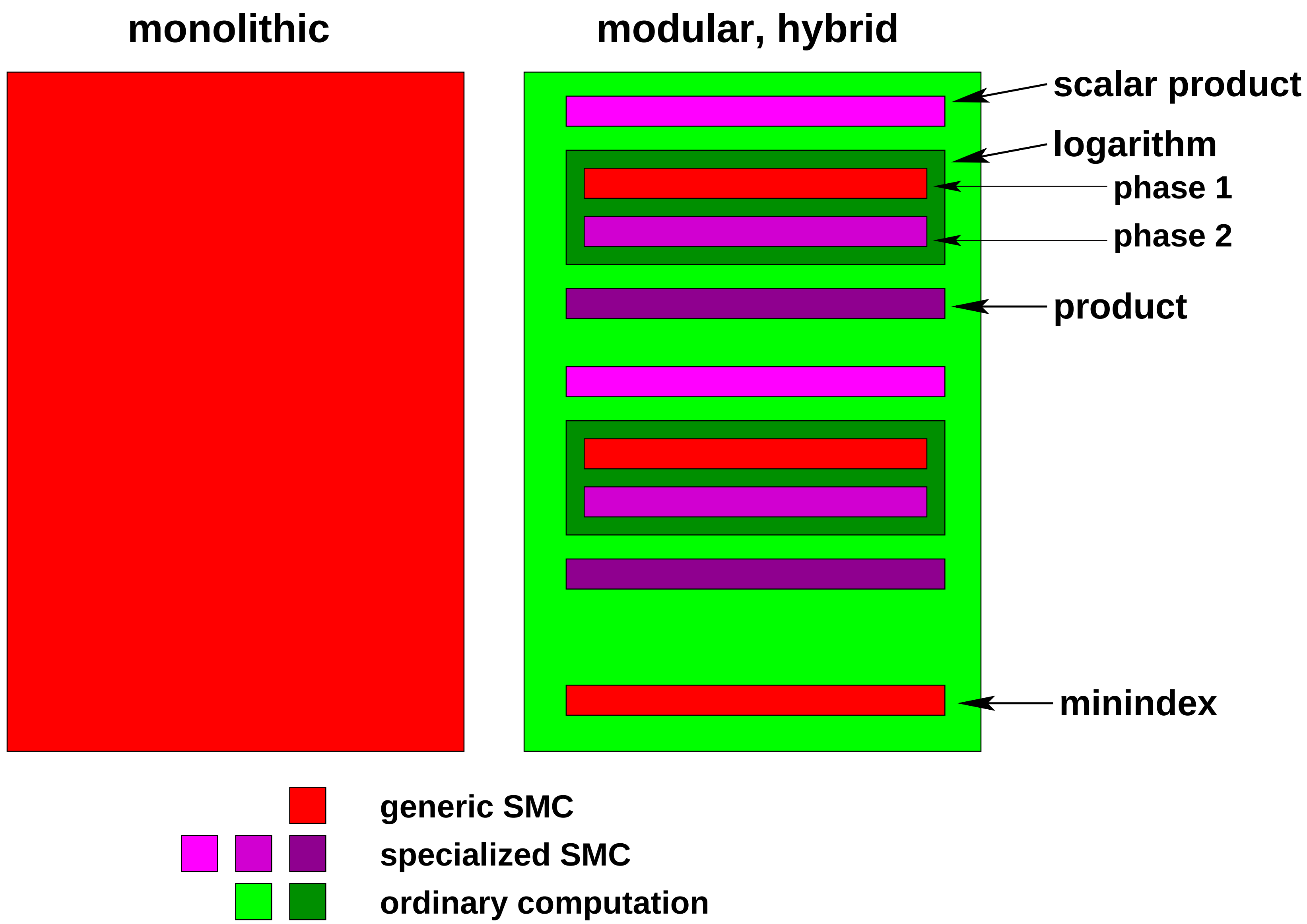
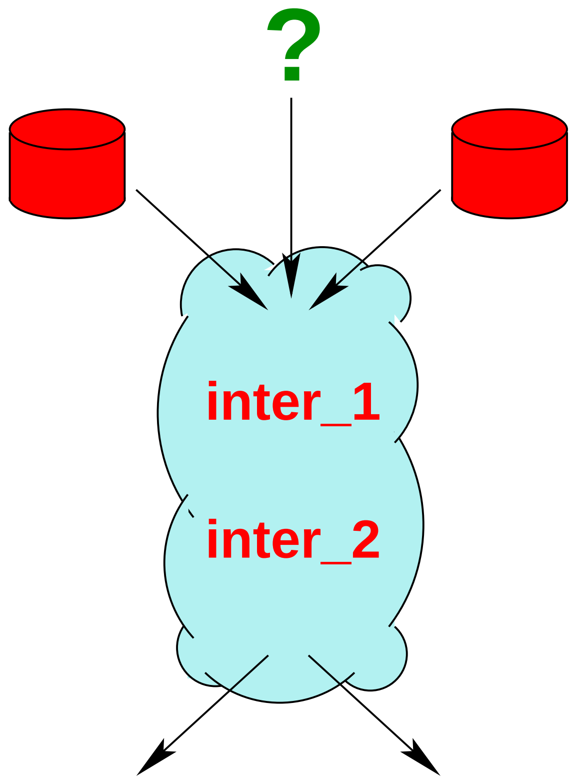
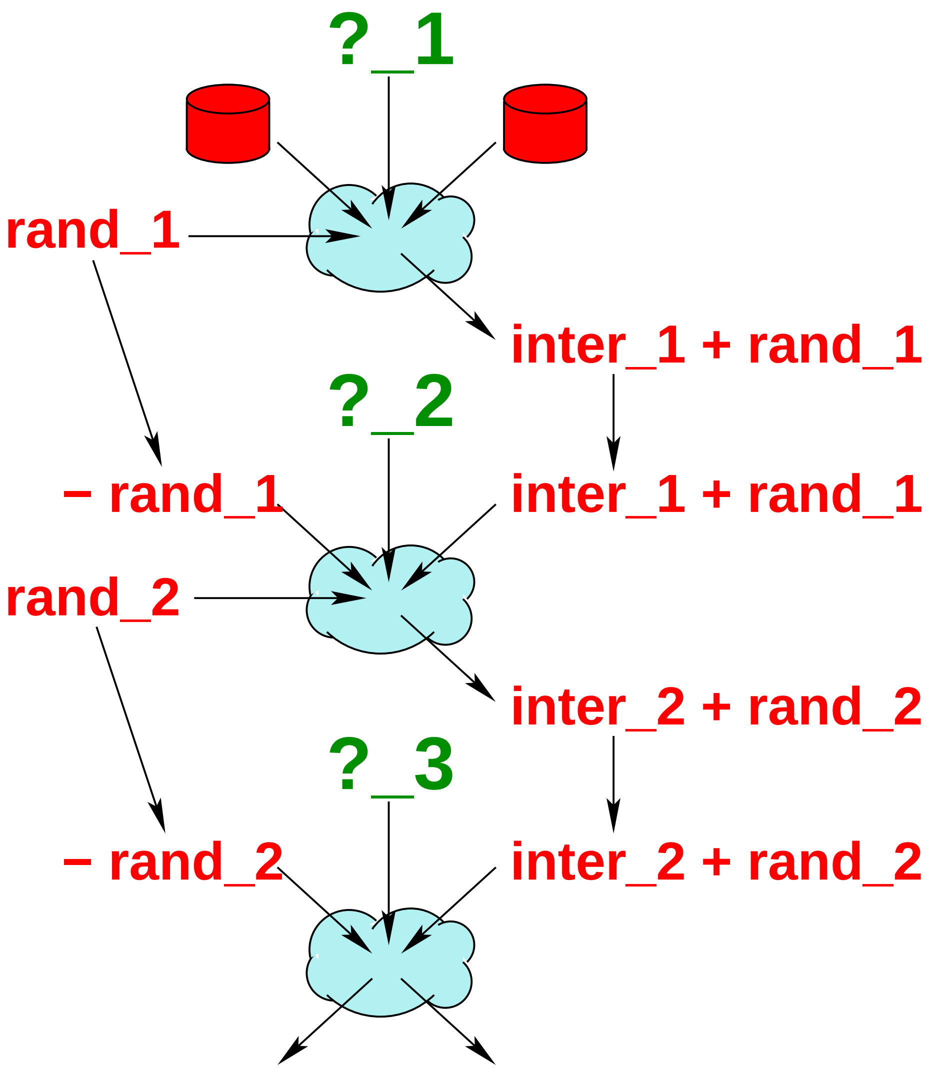
Monolithic vs. modular SMC



Shares to shares: the key to modularity with security





Toward the Lindell-Pinkas theses in practice

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Using homomorphic encryption:

- ☐ Private bit vectors to private shares of their **scalar product**.
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- Sequences of private shares of a sequence of values to their (public) **minindex**, the (smallest) index of the minimum.

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Using the Yao generic two-party SMC scheme:

- ☐ Sequences of private shares of a sequence of values to their (public) **minindex**, the (smallest) index of the minimum.

... And using both the Yao generic scheme and homomorphic encryption:

- ☐ Private shares of an argument to private shares of its **logarithm**, following the Lindell-Pinkas proposal—corrected, optimized, and implemented in the work presented here.

The Lindell-Pinkas $\ln x$ protocol: overall plan

- Multiplicatively decompose x as $2^n(1+\varepsilon)$, where $-1/4 \leq \varepsilon < 1/2$. Additively decompose the logarithm,

$$\ln x \;=\; \ln 2^n(1+\varepsilon) \;=\; n\ln 2 + \ln(1+\varepsilon) \qquad (1)$$

  The Taylor expansion of the latter term,

$$\ln(1+\varepsilon) \;=\; \sum_{i=1}^{\infty} \frac{(-1)^{i-1}\varepsilon^i}{i} \;=\; \varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \frac{\varepsilon^4}{4} + \cdots \qquad (2)$$

  will allow **configurable accuracy**.
- Protocol phase 1: From shares of x, compute shares of n and ε using **generic Yao** two-party secure computation.
- Protocol phase 2: From the shares of ε yielded by phase 1, compute shares of $\ln(1+\varepsilon)$—to "enough" terms of its expansion—using **oblivious polynomial evaluation**.

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How many bits of precision?

- **Must** be decided in advance!
- Let N be the lowest agreed upper bound on n. ε may have as many as N bits of precision, which we want to preserve.
- We want similar precision in the output.
- Therefore, since we will be computing in integers, the polynomial we compute in phase 2 must be adjusted to accept ε scaled up by 2^N; and to deliver $\ln(1+\varepsilon)$ scaled up by some factor σ that should be at least 2^N.
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Accommodating the scaling in phase 2

- Where α_1 and α_2 are the parties' respective additive shares, in some finite field (or ring) $\mathcal{F}$, of $\varepsilon \cdot 2^N$ to be delivered by phase 1,

$$\varepsilon = (\alpha_1 +_{\mathcal{F}} \alpha_2)/2^N$$

- Scaling the phase 2 output up by factor σ, the Taylor series of (2) becomes

$$\sigma \ln(1+\varepsilon) = \sum_{i=1}^{\infty} \frac{\sigma(-1)^{i-1}(\alpha_1 +_{\mathcal{F}} \alpha_2)^i}{i\, 2^{Ni}}$$

- ... But we will need a finite polynomial over $\mathcal{F}$ for the oblivious polynomial evaluation.

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From Taylor series over $\mathbb{R}$ to polynomial over $\mathcal{F}$

- Truncate the series at k terms for the desired accuracy.
- **If** the numerator will always be divisible by the denominator (in $\mathbb{Z}$); and ...
- **if** we use an $\mathcal{F}$ large enough so that, where $m = |\mathcal{F}|$, all values in the recursive evaluation are always integers in the interval $[-\lfloor \frac{m}{2} \rfloor, \lfloor \frac{m}{2} \rfloor]$; ...
- **then** we can reinterpret the additions and multiplications, and even the divisions, as the corresponding operations in $\mathcal{F}$, ...
- allowing us to replace 'α_2' with variable 'y', then open parentheses and collect terms to arrive at a polynomial over $\mathcal{F}$ for oblivious polynomial evaluation.

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Setting the scale-up: the original Lindell-Pinkas version

- Lindell and Pinkas set the scale-up factor σ at $2^N \operatorname{lcm}(2,\ldots,k)$, giving the truncated Taylor series

$$\ln(1+\varepsilon)\cdot 2^N \operatorname{lcm}(2,\ldots,k) \approx \sum_{i=1}^{k} \frac{(-1)^{i-1}\,(\operatorname{lcm}(2,\ldots,k)/i)\,(\alpha_1 +_{\mathcal{F}} \alpha_2)^i}{2^{N(i-1)}}$$

- In the numerator,

$$(\alpha_1 +_{\mathcal{F}} \alpha_2)^i = (\varepsilon \cdot 2^N)^i = \varepsilon^i \cdot 2^{Ni}$$

- Yet this is **not** generally divisible by $2^{N(i-1)}$.

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Arbitrary scaling: naive Yao recourse

- Scaling **up** by an **integer** factor:
  autonomously by the parties, no problem.
- Scaling **down** by an integer factor, or, more generally, scaling by a **non-integer** factor:
  requires an SMC episode.
- Autonomous scaling by a non-integer factor is not possible—even to integer approximation! **Approximate division does not distribute over modular addition.**
- A Yao SMC episode can accomplish arbitrary scaling, but division and table look-ups are expensive.

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- Integer part of scale-up factor σ handled separately, leaving a scale-**down** to compute and add modularly.
- For p parties, only p variants of excess in the simple distribution of the scale-down over p original shares.
- A Yao circuit can
  - accept the parties' original shares;
  - accept the parties' simple-minded autonomous scale-downs;
  - accept a random value from parties 1 through $p-1$;
  - determine from the non-modular sum of the original shares which correction to apply to the autonomous scale-downs, and share the corrected scale-down using the random values.

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- Theoretically challenging.
- Eminently practical.

Benefits for the Lindell-Pinkas logarithm protocol

- Compatibility:
  We can efficiently **reverse unwanted scale-ups** that have entered as technical artifacts.

- Performance:
  We can efficiently **achieve wanted scale-ups**, and so avoid the **table look-up** recommended by Lindell and Pinkas to convert n to $2^N \cdot n \ln 2$ **within the Yao computation** of phase 1.

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- Bignums and basic cryptographic math from libssl and libcrypto.

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- Intel Pentium M at 1.86 GHz.

N	k	modulus bits	gates	absolute error	time (seconds)
13	4	60	1386	< 0.00458	3.57
22	5	120	2797	< 0.00183	6.16
28	7	210	4732	< 0.00034	10.04

Conclusion

- The Lindell-Pinkas two-party secure logarithm protocol, as it has evolved in the course of our implementation, seems to work well and be quite usable as a module in a complex two-party SMC data-mining protocol.
- SMC usability and performance enhancements will continue.
- ... But SMC can already do much now. The main impediment to real-world application is a **gap in awareness and understanding** of what can already be done with SMC today, a gap that is just beginning to be addressed.