CPSC 468/568: Exam 1

October 15, 2009

Answer all five of the following questions. Please remember to write your name, CPSC 468/568, and today's date on the covers of all blue books you submit.

Question 1

An instance of the *Generalized Kayles* problem consists of an undirected graph $G = (V, E)$. The instance represents a game in which Players 1 and 2 alternate moves. Player 1 moves first. A move consists simply of choosing a vertex; when a vertex x is chosen by a player, x and all of its neighbors are removed from G, as are all edges adjacent to at least one of the vertices just removed. Player 1 wins if and only if Player 2 is the first player left with no vertices to choose. A yes instance of Generalized Kayles is one in which Player 1 has a forced win.

(a) (3 points) Give a yes instance of Generalized Kayles.

(b) (3 points) Give a no instance of Generalized Kayles.

(c) (14 points) Prove that Generalized Kayles is in PSPACE.

Question 2

(a) (10 points) Recall the following special case of the Gap Theorem: For any total, computable function g such that $g(n) \geq n$, for all n, there exists a time bound $T(n)$ such that $\text{DTIME}(g(T(n))) = \text{DTIME}(T(n))$. Prove that, for $g(n) = n^2$, the function T is not time-constructible.

(b) (10 points) Define the complexity classes $\text{ATIME}(T(n))$ and $\text{ASPACE}(S(n))$.

Question 3

(a) (10 points) Why are time- and space-complexity classes such as $\text{DTIME}(T(n))$ and $\text{DSPACE}(S(n))$ defined only up to constant factors?

(b) (10 points) Why does the existence of an oracle that separates two complexity classes (*e.g.*, the existence of an oracle B such that $P^B \neq NP^B$) not prove that the two classes are different (*resp.*, not prove that $P \neq NP$)?

Question 4

State whether each of the following claims is true, false, or unknown. If you answer true or false, give a very brief justification.

(a) (5 points) Every unary language is decidable.

(b) (5 points) TQBF is PH-Complete.

(c) (5 points) NP = P/poly

(d) (5 points) PATH is coNL-complete.

Question 5

(a) (10 points) Let DISCONN be the set of directed graphs that are not strongly connected. Show that DISCONN is in NL.

(b) (10 points) This question asks you to explain one step in the proof of the Karp-Lipton Theorem. Recall that the theorem tells us that, if NP $\subseteq$ P/poly, then the PH collapses at the second level. The proof consists of showing that, if NP $\subseteq$ P/poly, then Π_2SAT, which consists of all true formulas of the form

$$\forall u \in \{0,1\}^n \; \exists v \in \{0,1\}^n \; \phi(u,v) = 1 \tag{1}$$

is in Σ_2^P; here, ϕ is a quantifier-free boolean formula. The first step in the proof is to note that, for every string u, the "$\exists v \in \{0,1\}^n \phi(u,v) = 1$" part of (1) is a SAT instance. Thus, if NP $\subseteq$ P/poly, there is a polynomial-sized circuit family $\{C_n\}_{n\geq 1}$ such that (1) is equivalent to:

$$\forall u \in \{0,1\}^n \; C_n(\phi, u) = 1. \tag{2}$$

The second step in the proof is to notice that there is another circuit family (say $\{C'_n\}_{n\geq 1}$, where C'_n can be encoded by a string of at most $w(n)$ bits and $w()$ is a polynomial) such that, if NP $\subseteq$ P/poly, then (1) is equivalent to

$$\exists C'_n \in \{0,1\}^{w(n)} \; \forall u \in \{0,1\}^n \; \phi(u, C'_n(\phi, u)) = 1. \tag{3}$$

Why is the second step valid?