A Turing-Machine model of Computation

Deterministic $k$-tape Turing machine $M$.

There is one read-only input tape (on top) and $k-1$ read-write work/output tapes. $M$ is a triple $\Gamma, Q, \delta$ that is defined as follows:

- $\Gamma$ is the tape alphabet, a finite set of symbols. Assume $\Box$ ("blank" symbol), $\triangleright$ ("start" symbol), 0 and 1 are four distinct elements of $\Gamma$.

- $Q$ is the state set, a finite set of states that $M$’s control register can be in. Assume $q_{\text{start}}$ and $q_{\text{halt}}$ are two distinct states in $Q$.

- $\delta$ is the transition function, a finite table that describes the rules (or program) by which $M$ operates:

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^{k-1} \times (L, S, R)^k.$$
\[ \delta(q, (\sigma_1, ..., \sigma_k)) = (q', (\sigma'_2, ..., \sigma'_k), (z_1, ..., z_k)) \] means that, if \( M \) is in state \( q \), and the read (or read/write) tape heads are pointing at the cells containing \( \sigma_1, ..., \sigma_k \), then the following “step” of the computation is performed:

- the read/write tape symbols \( \sigma_2, ..., \sigma_k \) are replaced by \( \sigma'_2, ..., \sigma'_k \);
- tape head \( i \) moves left, stays in place or moves right, depending on whether \( z_i \) is in \( L, S \) or \( R \);
- the control-register state is changed to \( q' \).

When \( M \) starts its execution on input \( x = \sigma_1, ..., \sigma_n \), we have

- \( q = q_{\text{start}} \)
- input tape

\[ \triangleright \sigma_1 \sigma_2 \cdots \sigma_n \square \square \cdots \]

- all other tapes

\[ \triangleright \square \square \square \square \square \square \cdots \]

Meaning of \( q_{\text{halt}} \):

\[ \delta(q_{\text{halt}}, (\sigma_1, ..., \sigma_k)) = (q_{\text{halt}}, (\sigma_2, ..., \sigma_k), S^k) \quad \forall (\sigma_1, ..., \sigma_k). \]

Designate one of the read/write tapes as "the output tape".

Turing machine \( M \) "computes the function \( f' \)", if for all \( x \in \Gamma^* \) the execution of \( M \) on input \( x \) eventually reaches the state \( q_{\text{halt}} \), and when it does, the contents of \( M \)'s output tape is \( f(x) \).

\( M \) "runs in time \( T' \)" if for all \( n \) and all \( x \in \Gamma^n \) \( M \) halts after at most \( T(n) \) steps.