Computational Complexity

CPSC 468/568, Fall 2012
Time: Tu & Th, 2:30-3:45 pm
Room: AKW 000
Satisfies the QR requirement.

http://zoo.cs.yale.edu/classes/cs468/
Partial Topic Outline

- Complexity classes (P, NP, L, NL, etc.)
- Reductions and completeness
- The roles of, e.g.,
  - Randomness
  - Interaction
  - Approximation

? Communication complexity
Requirements (undergrads)

- 6 Written HW Assignments, each worth 10% of the course grade
- 2 In-Class Exams, each worth 20% of the course grade
- No final exam during exam week
Requirements (grad students)

- 6 Written HW Assignments, each worth 9% of the course grade
- Lecture-note help, worth 6% of the course grade
- 2 In-Class Exams, each worth 20% of the course grade
- No final exam during exam week
Tentative Schedule

Sept. 18: First HW Assignment Due
Sept. 27: Second HW Assignment Due
Oct. 9: Third HW Assignment Due
Oct. 11: First In-Class Exam
Oct. 19: Fall Semester Drop Date
Nov. 1: Fourth HW Assignment Due
Nov. 15: Fifth HW Assignment Due
Dec. 4: Sixth HW Assignment Due
Dec. 6: Second In-Class Exam
Rules and Guidelines

• Deadlines are firm.
• Late penalty: 5% per day for at most 7 days, after which solutions are posted and HWs not yet turned in receive a grade of zero.
• Announcements and assignments will be posted on the class webpage (as well as conveyed in class).

➢ No “collaboration” on homeworks unless you are told otherwise.
• Pick up your graded homeworks and exams promptly, and tell the TA promptly if one is missing.
Instructor: Joan Feigenbaum
Office: AKW 512
Office Hours: Thursdays 10:30 am - 12:30 pm and by appointment
Phone: 203-432-6432
Assistant: Judi Paige
(judi.paige@yale.edu, 203-436-1267, AKW 507a, 8:30 am - 4:30 pm M-F)

Note: Do not send email to Professor Feigenbaum, who suffers from RSI. Contact her through Ms. Paige or the TA.
TA: Aaron Segal
Office: AKW 503
Email: Aaron.Segal@yale.edu
Office Hours:
  Tues and Thurs, 4 - 5 pm
  and by appointment
If you’re undecided …

Check out:

• zoo.cs.yale.edu/classes/cs468/fall10/
• www.cs.princeton.edu/theory/complexity/
  (draft of textbook by Sanjeev Arora and Boaz Barak of Princeton)
• www.cs.berkeley.edu/~luca/cs278-02/
  (a complexity-theory course taught by Luca Trevisan at Berkeley in 2002)
• www.cs.lth.se/home/Rolf_Karlsson/bk/retro.pdf
  (“NP-Completeness: A Retrospective,” by Christos Papadimitriou, 1997 International Colloquium on Automata, Languages, and Programming)
Questions?
Introduction to Complexity Classes
Computational Complexity
Themes

• “Easy” vs. “Hard”
• Reductions (Equivalence)
• Provability
• Randomness
Poly-Time Solvable

- Nontrivial Example: Matching
Poly-Time Solvable

• Nontrivial Example: Matching
Poly-Time Verifiable

- Trivial Example: Hamiltonian Cycle
Poly-Time Verifiable

• Trivial Ex. : Hamiltonian Cycle
• Is it Easier to Verify a Proof than to Find one?

• Fundamental Conjecture of Computational Complexity: $P \neq NP$
• Matching:

• HC:

Fundamentally Different
Reduction of $B$ to $A$

• If $A$ is “Easy,” then $B$ is, too.
• NP-completeness
• P-time reduction
• Cook’s theorem
  \[ \text{If } B \in \text{NP}, \text{ then } B \leq_{\text{P-time}} \text{SAT} \]
• HC is NP-complete
Equivalence

• **NP-complete** problems are an equivalence **Class** under polynomial-time reductions.
• 10k’s problems
• Diverse fields

  Math, CS, Engineering, Economics, Physical Sci., Geography, Politics...
\[ \text{NP} \cap \text{coNP} = \text{P} \]
Random poly-time Solvable

\( x \in \{0, 1\}^n \)
\( r \in \{0, 1\}^{\text{poly}(n)} \)
Probabilistic Classes

\[ \begin{align*}
\text{RP} & : x \in L \rightarrow \text{“yes” w.p. } \frac{3}{4} \\
 & \quad x \notin L \rightarrow \text{“no” w.p. 1}
\end{align*} \]

\[ \begin{align*}
\text{coRP} & : x \in L \rightarrow \text{“yes” w.p. 1} \\
 & \quad x \notin L \rightarrow \text{“no” w.p. } \frac{3}{4}
\end{align*} \]

(Outdated) Nontrivial Result

PRIMES \in \text{ZPP (} = \text{RP} \cap \text{coRP})
Two-sided Error

\[
\begin{align*}
\text{BPP} & \quad \begin{cases} 
    x \in L \rightarrow \text{“yes”} \quad \text{w.p. } \frac{3}{4} \\
    x \notin L \rightarrow \text{“no”} \quad \text{w.p. } \frac{3}{4}
\end{cases}
\end{align*}
\]

Question to Audience: BPP set not known to be in RP or coRP?

Note: QR is in NP and coNP but not known to be in RP or coRP.
Interactive Provability

$P \times V \{ \text{PPT, } \phi \}$
$L \in IP$

- $x \in L \rightarrow \exists P: \text{“yes” w.p. } \frac{3}{4}$
- $x \notin L \rightarrow \forall P^*: \text{“no” w.p. } \frac{3}{4}$

Nontrivial Result

Interactively Provable

Poly-Space Solvable