

Computer Science 468/568  
Homework #1, due in class Tuesday, January 23.

Reading: Introduction and Chapters 0, 1 and 2 of Arora and Barak, “Computational Complexity: A Modern Approach.” Much of this reading should be review of material you have already seen in previous classes. The textbook has a number of (generally minor) typos – good practice for reading theory papers.

1. (10 points) Please list any persons (including course staff) and/or resources you consulted in connection with this assignment, including full citations for resources, (that is, enough information for another person to find them).
2. (10 points) The course will cover most of Chapters 1-8 and 11 in the textbook. Please select two additional chapters from the textbook that interest you, specify which they are and give a cogent argument for each indicating why it (or part of it) should be covered in the course.
3. (20 points) Give the definition of what it means for a Turing machine to be oblivious. Modify the Turing machine described in Example 1.1 of the textbook to be oblivious and show that it is.
4. (30 points) ThinkFun sells a puzzle game called Code Master, part of which can be generalized as follows. The input is a directed graph  $G$  with a designated start node and a designated end node. Each edge is assigned one of  $c$  colors (represented by integers  $1, \dots, c$ ). For each node, there is at most one edge of each color directed out of it. Each node may or may not have a power crystal on it. You are also given  $k_i$  tokens of color  $i$ , for  $i = 1, \dots, c$ . Your task is (if possible) to find a linear ordering of the tokens such that starting at the start node and following the out-going edge of the color corresponding to the next token in sequence, you end at the end node, having collected all the power crystals from the graph and having used up exactly all the tokens. (You collect a power crystal from a node when you first arrive at it.)

Prove that the decision version of this problem is NP-complete. What if there are only two colors ( $c = 2$ )? What if the graph is undirected? What if there are no power crystals? (For full credit you should address the original problem and at least one variant (imposing one or more of the three conditions).)

5. (30 points) Problem 2.30 in the textbook: Show that if there is a unary NP-complete language, then  $P=NP$ . (Note that H532 after the problem means there is a hint on page 532 of the textbook.)