Computer Science 468/568
Homework #3, due in class Thursday, February 8, 2018.

1. (10 points) Please list any persons (including course staff) you talked with about this assignment and cite any resources (other than the textbook) you consulted in connection with this assignment (with enough information that another person could find the materials you cite).

2. (30 points) Let C be the set of all strings α such that the Turing machine M_α runs in polynomial time. Let HALT be the set of all pairs of strings $\langle \alpha, x \rangle$ such that M_α halts in a finite number of steps on input x. Show that C is undecidable by proving HALT $\leq_p C$.

3. (30 points) If L_1 and L_2 are languages, let

$$L_1 \oplus L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1).$$

   (a) Assume that $L_1 \in \text{DTIME}(n^2)$ and $L_2 \in (\text{DTIME}(n^3) \setminus \text{DTIME}(n^2))$. Prove that $L_1 \oplus L_2$ cannot be a finite set.

   (b) Prove that there exists a language $L_2 \in \text{DTIME}(n^3)$ such that for every $L_1 \in \text{DTIME}(n^2)$ there exists a natural number n_0 such that $L_1 \oplus L_2$ contains at least one string of length n for every $n \geq n_0$.

4. (30 points) Prove that there exist languages L and A such that $L \in \text{NP}^A$ but L is not polynomial time reducible to SAT, even if the Turing machine performing the reduction has oracle access to A. (Hint: try to construct A by diagonalizing over potential polynomial time reductions that have oracle access to A.)

5. (30 points) This problem is required of students enrolled in CPSC 568, but not students enrolled in CPSC 468. Students enrolled in CPSC 468 may do the problem and have it graded, but will receive no extra credit for it.

   Read the Wikipedia page on the topic of NP-intermediate problems. Select one of the problems from the list (other than factoring) and read some reference material about the problem (don't forget to cite it) and then in your own words carefully define the problem and explain why people think it might be an NP-intermediate problem.

   The page is: https://en.wikipedia.org/wiki/NP-intermediate.