Computer Science 468/568
Homework #4, due in class Thursday, February 15, 2018.

1. (10 points) Please list any persons (including course staff) you talked with about this assignment and cite any resources (other than the textbook) you consulted in connection with this assignment (with enough information that another person could find the materials you cite).

2. (30 points) For any positive integer k, define SC^k to be the class of all languages L such that there exists a Turing machine M to decide L that runs both in polynomial time and in space $O((\log n)^k)$. Let SC be the union of SC^k for all positive integers k. Steve Cook, for whom the class is named, showed that deterministic context free languages are in SC^2.

   Note that it is unknown whether PATH is in SC. Explain why Savitch's Theorem (Theorem 4.14) doesn't settle this question.

   What happens to SC if P is equal to SPACE($\log n$)? Prove your answer.

3. (30 points) If $x \in \{0,1\}^*$ we define $\text{square}(x) = x^n$, where $n = |x|$. That is, square(x) is the concatenation of x with itself $|x|$ times. We lift square to languages in the usual way: $\text{square}(L) = \{\text{square}(x) : x \in L\}$.

   Prove the following.

   (a) For all languages L, $L \in \text{P}$ iff $\text{square}(L) \in \text{P}$.
   (b) There exists a language L such that $L \notin \text{SPACE}(n)$ and $\text{square}(L) \in \text{SPACE}(n)$.

   Explain what you can conclude about P and SPACE(n) as a result.

4. (30 points) We consider a 2-player version of Code Master. The board is a directed graph of n nodes with a designated start node, a designated end node, and a coloring of the edges. Two players, P_1 and P_2, are given multisets S_1 and S_2 of at most n^2 colored tokens each.

   The game is played as follows. A unique movable marker is placed on the start node. Beginning with P_1, the players take turns spending one of their tokens to move the marker along an outgoing edge to the node at the end of the edge. The color of the token spent must match the color of the edge chosen. If there is no possible move, either because the player is out of tokens, or none of their tokens match the color of any of the outgoing edges of the node the marker is on, then P_2 wins. If there is any possible move, the player must make one of the possible moves. Player P_1 wins if the marker ever arrives at the end node.

   The problem we consider is: given the board and the multisets of tokens of the two players, is there a winning strategy for P_1? Prove that this problem is PSPACE-complete. (Hint: Generalized Geography might be related.)

5. (30 points) This problem is required of students enrolled in CPSC 568, but not of students enrolled in CPSC 468. Students enrolled in CPSC 468 may do the problem and have it graded, but will receive no extra credit for it.

   If G is an undirected graph, its *girth* is the length of the smallest simple cycle in G. Prove that the language containing all pairs $\langle G, k\rangle$ such that G is an undirected graph of girth at least k is in NL. Assume that G is represented by its adjacency matrix and k is represented in binary.