Computer Science 468/568
Homework #5, due in class Thursday, February 22, 2018.

1. (10 points) Please list any persons (including course staff) you talked with about this assignment and cite any resources (other than the textbook) you consulted in connection with this assignment (with enough information that another person could find the materials you cite).

2. (30 points) The class **DP** is defined as the set of languages L for which there are two languages $L_1 \in \mathbf{NP}$ and $L_2 \in \mathbf{coNP}$ such that $L = L_1 \cap L_2$. (Do not confuse **DP** with $\mathbf{NP} \cap \mathbf{coNP}$, which may seem superficially similar.) Show that

   (a) EXACT-INDSET is in **DP**, where EXACT-INDSET is the set of all $\langle G, k\rangle$ such that G is an undirected graph and k is the size of the largest independent set in G.

   (b) Every language in **DP** is polynomial time reducible to SAT-UNSAT, where SAT-UNSAT is the set of all $\langle \phi, \psi \rangle$ such that ϕ and ψ are 3CNF formulas, ϕ is satisfiable, and ψ is unsatisfiable.

3. (30 points) Let **AL** be the class of languages decidable by alternating log space bounded Turing machines. Below we'll prove $\mathbf{AL} = \mathbf{P}$.

   (a) Show that the ACCEPT marking procedure described in the text can be implemented to show that $\mathbf{AL} \subseteq \mathbf{P}$.

   (b) Let $L \in \mathbf{P}$. Without loss of generality, we can assume that L is decided by a one-tape Turing machine running in time less than $n^c + k$ for some constants c and k – BRIEFLY explain why. Given an input $x \in \{0,1\}^n$, we can think of the configurations of M on input x as represented by a square array C of $T = n^c + k$ rows of tape symbols, with row t giving the tape contents at (the start of) step t, together with $q[t]$, the state at step t, and $h[t]$, the head position at step t. Of course, $O(\log n)$ space is not enough to write down even one such configuration in general.

   In general, the correctness of the state $q[t]$, the head position $h[t]$ and the contents of any tape square $C[t, j]$ depend only on the correctness of the previous state $q[t-1]$, the previous head position $h[t-1]$, the previously scanned symbol $C[t-1, h[t-1]]$, and the previous contents of square j, $C[t-1, j]$.

Using alternation and space $O(\log n)$, we can check whether M accepts x by starting at the end of the computation, guessing a step t between 1 and T, an accepting state $q[t]$ and a head position $h[t]$, and then trying to verify that these are correct for M's computation. To do so, we guess the previous state $q[t-1]$, the previous head position $h[t-1]$, and the previously scanned symbol $C[t-1, h[t-1]]$, check that these correctly lead to the values at step t, and then verify that all the guessed values are correct (which means going back another step ...) When $t = 1$, we can directly verify the state is the start state, the head position is 1, and the tape symbols are the symbols of x followed by blanks.

Develop this outline into a careful proof that $\mathbf{P} \subseteq \mathbf{AL}$.

4. (30 points) This problem is required of students enrolled in CPSC 568, but not of students enrolled in CPSC 468. Students enrolled in CPSC 468 may do the problem and have it graded, but will receive no extra credit for it.

   Look at the list compiled by Marcus Schaefer and Christopher Umans entitled *Completeness in the Polynomial-Time Hierarchy: A Compendium*, which is available at: http://ovid.cs.depaul.edu/documents/phcom.pdf.

   Pick out a problem relevant to your research (or other) interest and give a careful definition of the problem and a clear explanation of what is known about its computational complexity. Have there been any results regarding the problem since 2008?