Computational Complexity

CPSC 468/568, Spring 2016
Time: Tu & Th, 2:30-3:45 pm
Room: AKW 000

http://zoo.cs.yale.edu/classes/cs468/
Partial Topic Outline

• Complexity classes (P, NP, L, NL, etc.)
• Reductions and completeness
• The roles of, e.g.,
  - Randomness
  - Interaction
  - Approximation

? Communication complexity
Requirements (undergrads)

• 6 Written HW Assignments, each worth 10% of the course grade
• 2 In-Class Exams, each worth 20% of the course grade
• No final exam during exam week
Requirements (grad students)

• Modest reading assignments, mostly in Arora and Barak, *Computational Complexity: A Modern Approach*.

• 6 Written HW Assignments, each worth 9% of the course grade

• Note taking and/or lecturing, worth 6% of the course grade

• 2 In-Class Exams, each worth 20% of the course grade

• No final exam during exam week
Tentative Schedule

Thurs, February 4: First HW Assignment Due
Thurs, February 18: Second HW Assignment Due
Thurs, March 3: Third HW Assignment Due
Tues, March 8: First In-Class Exam
Fri, March 11: Spring Semester Drop Date
Thurs, March 31: Fourth HW Assignment Due
Tues, April 12: Fifth HW Assignment Due
Tues, April 26: Sixth HW Assignment Due
Thurs, April 28: Second In-Class Exam
Rules and Guidelines

• Deadlines are firm.
• Late penalty: 5% per day for at most 7 days, after which solutions are posted and HWs not yet turned in receive a grade of zero.
• Announcements and assignments will be posted on the class webpage (as well as conveyed in class).

➢ Try to do the HW on your own. If you work in a group to solve a HW problem, identify the group members on your HW paper. If you use any sources except the textbook and classnotes, identify them.

• Pick up your graded homeworks and exams promptly, and tell the TA promptly if one is missing.
Instructor: Joan Feigenbaum  
Office: AKW 512  
Office Hours: TBA and by appointment  
Phone: 203-432-6432  
Assistant: Judi Paige  
(judi.paige@yale.edu, 203-436-1267, AKW 507a, 8:30 am – 4:30 pm M-F)  

Note: Do not send email to Professor Feigenbaum, who suffers from RSI.  
Contact her through Ms. Paige or the TA.
TA: Debayan Gupta
Office: AKW 503
Email: Debayan.Gupta@yale.edu
Office Hours:
  Tues and Thurs, 4 - 5 pm
  and by appointment
If you’re undecided ...

Check out:

* [http://zoo.cs.yale.edu/classes/cs468/spr15/](http://zoo.cs.yale.edu/classes/cs468/spr15/)
Questions?
Introduction to Complexity Classes
Computational Complexity Themes

• “Easy” vs. “Hard”
• Reductions (Equivalence)
• Provability
• Randomness
Poly-Time Solvable

Nontrivial Example: Matching
Poly-Time Solvable

Nontrivial Example: Matching
Poly-Time Verifiable

Trivial Example: Hamiltonian Cycle
Poly-Time Verifiable

Trivial Example: Hamiltonian Cycle
• Is it **Easier** to **Verify** a Proof than to **Find** one?

• **Fundamental Conjecture of Computational Complexity:**

\[
P \neq NP
\]
Distinctions

Matching:

HC:

Fundamentally Different
Reduction of \( B \) to \( A \)

If \( A \) is “Easy,” then \( B \) is, too.

“A” is the “oracle” or “black box” for \( B \) Algorithm.
• NP-completeness
• P-time reduction
• Cook’s theorem
  If $B \in \text{NP}$, then $B \leq_{\text{P-time}} \text{SAT}$
• HC is NP-complete
Equivalence

• NP-complete problems are an equivalence Class under polynomial-time reductions.
• 10k’s problems
• Diverse fields

Math, CS, Engineering, Economics, Physical Sci., Geography, Politics...
NP

coNP

P
Random poly-time Solvable

\[ x \in L? \]

poly-time Algorithm

\[ x \in \{0,1\}^n \]
\[ r \in \{0,1\}^{\text{poly}(n)} \]
Probabilistic Classes

RP
\[
\begin{align*}
x \in L &\rightarrow \text{“yes” } \text{ w.p. } \frac{3}{4} \\
x \notin L &\rightarrow \text{“no” } \text{ w.p. } 1
\end{align*}
\]

coRP
\[
\begin{align*}
x \in L &\rightarrow \text{“yes” } \text{ w.p. } 1 \\
x \notin L &\rightarrow \text{“no” } \text{ w.p. } \frac{3}{4}
\end{align*}
\]

(Outdated) Nontrivial Result
\[
\text{PRIMES } \in \text{ ZPP (} = \text{ RP } \cap \text{ coRP)}
\]
Two-sided Error

\[ \text{BPP} \begin{cases} 
  x \in L \rightarrow \text{“yes”} & \text{w.p. } \frac{3}{4} \\
  x \notin L \rightarrow \text{“no”} & \text{w.p. } \frac{3}{4}
\end{cases} \]

**Question to Audience:** Is there a BPP set not known to be in RP or coRP?

**Note:** QR is in NP and coNP but not known to be in RP or coRP.
Interactive Provability

$P \times V \ [\text{PPT, } \notin]$

yes/no
$L \in \text{IP}$

- $x \in L \rightarrow \exists P: \text{“yes” w.p. } \frac{3}{4}$
- $x \not\in L \rightarrow \forall P^*: \text{“no” w.p. } \frac{3}{4}$

**Nontrivial Result**

Interactively Provable

Poly-Space Solvable