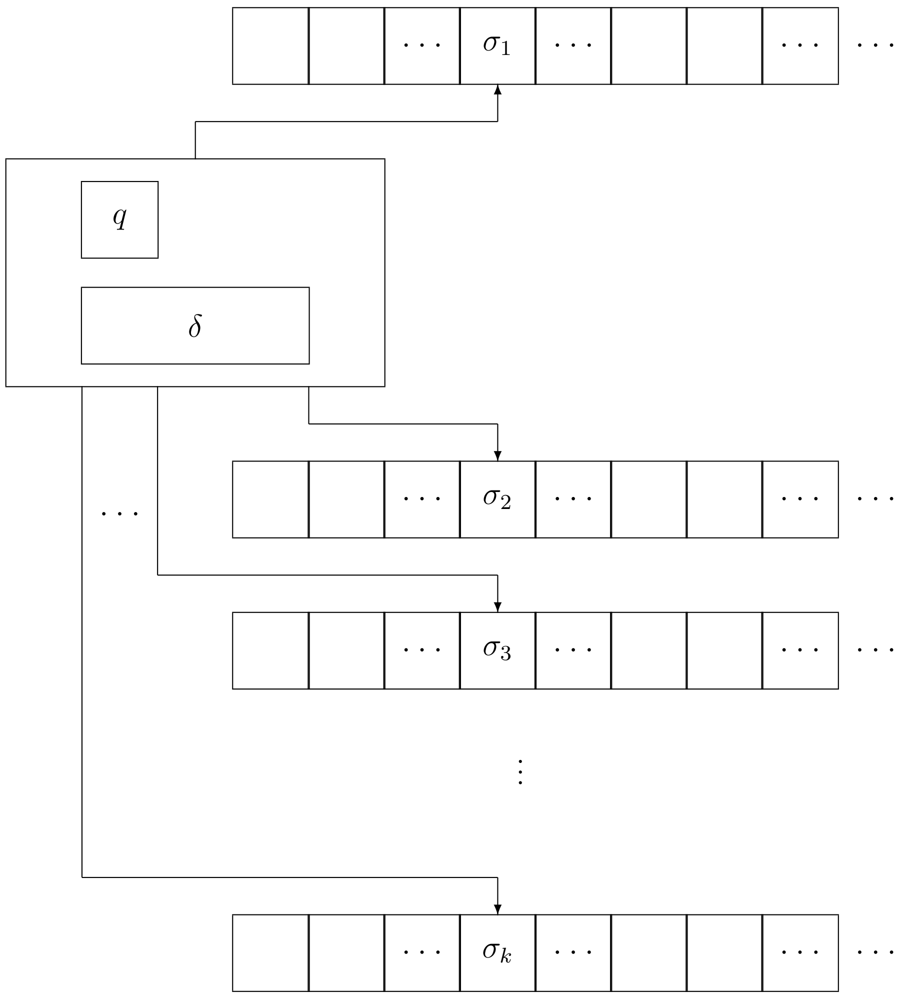
A Turing-Machine model of Computation

Deterministic k-tape Turing machine M.



There is one read-only **input tape** (on top) and $k-1$ read-write **work/output tapes**. M is a triple Γ, Q, δ that is defined as follows:

- Γ is the **tape alphabet**, a finite set of symbols. Assume $\Box$ ("blank" symbol), $\triangleright$ ("start" symbol), 0 and 1 are four distinct elements of Γ.
- Q is the **state set**, a finite set of states that M's control register can be in. Assume q_{start} and q_{halt} are two distinct states in Q.
- δ is the **transition function**, a finite table that describes the rules (or program) by which M operates:

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^{k-1} \times (L, S, R)^k.$$

$\delta(q, (\sigma_1, ..., \sigma_k)) = (q', (\sigma'_2, ..., \sigma'_k), (z_1, ..., z_k))$ means that, if M is in state q, and the read (or read/write) tape heads are pointing at the cells containing $\sigma_1, ..., \sigma_k$, then the following "step" of the computation is performed:

- the read/write tape symbols $\sigma_2, ..., \sigma_k$ are replaced by $\sigma'_2, ..., \sigma'_k$;
- tape head i moves left, stays in place or moves right, depending on whether z_i is in L,S or R;
- the control-register state is changed to q'.

When M starts its execution on input $x = \sigma_1, ..., \sigma_n$, we have

- $q = q_{\text{start}}$
- input tape

$\triangleright$	σ_1	σ_2	$\cdots$	σ_n	$\square$	$\square$	$\cdots$	$\cdots$

- all other tapes

$\triangleright$	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$	$\cdots$	$\cdots$

Meaning of q_{halt}:

$$\delta(q_{\text{halt}}, (\sigma_1, ..., \sigma_k)) = (q_{\text{halt}}, (\sigma_2, ..., \sigma_k), S^k) \qquad \forall(\sigma_1, ..., \sigma_k).$$

Designate one of the read/write tapes as "the output tape".

Turing machine M "computes the function f", if for all $x \in \Gamma^*$ the execution of M on input x eventually reaches the state q_{halt}, and when it does, the contents of M's output tape is $f(x)$.

M "runs in time T" if for all n and all $x \in \Gamma^n$ M halts after at most $T(n)$ steps.