Answer five of the following six questions. If you answer all six, the first five of your answers will be graded, and the sixth will be ignored. Please remember to write your name, CPSC 468/568, and today’s date on the covers of all blue books you submit.

**Question 1**
(a) (2 points) Let $A$ be the set of pairs $(\alpha, x)$ such that Turing Machine $M_\alpha$ halts in at most $|x|^3$ steps on input $x$. Is $A$ undecidable? Give a yes-or-no answer and justify it in one sentence.

(b) (3 points) Define the term *oblivious Turing Machine*.

(c) (15 points) Construct an oracle $O$ such that $\text{NP}^O \neq \text{coNP}^O$.

**Question 2**
For purposes of this question, use the fact that the IND-SET (independent set) problem is NP-complete. Prove that the following three problems are also NP-complete. Prove that the following three problems are also NP-complete.

(a) (5 points) VC (vertex cover): the set of all pairs $(G, k)$, where $G = (V, E)$ is a graph and there is a subset $V'$ of $V$ such that $|V'| \leq k$ and, for every edge $(u, v)$ in $E$, at least one of $u$ or $v$ is in $V'$.

(b) (5 points) SUBGRAPH-ISO (subgraph isomorphism): the set of all pairs $(G, H)$ such that $G$ contains a subgraph that is isomorphic to $H$. An isomorphism $f$ from graph $G_1$ to graph $G_2$ is a bijective mapping from $V(G_1)$ to $V(G_2)$ such that, for all $v_1$ and $v_2$ in $V(G_1)$, $(v_1, v_2) \in E(G_1)$ if and only if $(f(v_1), f(v_2)) \in E(G_2)$.

(c) (10 points) DOMINATING SET: the set of all pairs $(G, k)$ such that there is a subset $V'$ of $V(G)$ such that $|V'| \leq k$ and every vertex $v$ in $V(G) - V'$ is adjacent to at least one member of $V'$.

**Question 3**
(a) (7 points) Define *implicitly logspace-computable reductions*. Explain why they are needed in the study of nondeterministic logspace.

(b) (7 points) Define *read-once certificates*. Explain why they are needed in the study of nondeterministic logspace.

(c) (6 points) Prove that, for every space-constructible $s(n) \geq \log n$, $\text{NSPACE}(s(n))$ and $\text{coNSPACE}(s(n))$ are contained in $\text{DTIME}(2^{O(s(n))})$. 
Question 4
State whether each of the following claims is true, false, or unknown. If you answer true or false, give a very brief justification.

(a) (4 points) \( NP = \text{P/poly} \)

(b) (4 points) \( \text{PSPACE} = \text{NPSPACE} \)

(c) (4 points) \( \text{DSPACE}(n) = \text{NSPACE}(n) \)

(d) (4 points) \( \text{NSPACE}(n) = \text{coNSPACE}(n) \)

(e) (4 points) \( \text{TQBF is PH-complete} \).

Question 5
The \text{PSPACE}-complete language \text{GEOGRAPHY} is defined as follows. An instance consists of a directed graph \( G = (V, E) \) and a designated start node \( s \in V \). Player I moves first by choosing node \( s \); then player II moves by choosing a node \( s' \neq s \) such that \( (s, s') \in E \). More generally, after \( m \) moves have been made, exactly \( m \) nodes have been chosen, and one of the two players has chosen node \( u \) in the \( m^{th} \) move; the \((m+1)^{st}\) move is then made by the other player, who must choose a node \( v \) such that \( (u, v) \in E \), and \( v \) has not already been chosen in one of the first \( m \) moves. When a player is unable to move (because no such node \( v \) exists), he loses. The instance \((G, s)\) is a yes-instance of \text{GEOGRAPHY} if and only if player I has a winning strategy.

(a) (3 points) Construct a yes-instance of \text{GEOGRAPHY}.

(b) (3 points) Construct a no-instance of \text{GEOGRAPHY}.

(c) (14 points) Prove that \text{GEOGRAPHY} is in \text{PSPACE}.

Question 6
(a) (3 points) Define the complexity class \text{DP}.

(b) (3 points) Which of the following relationships is known?
   (i) \text{DP} is contained in \( \text{NP} \cup \text{coNP} \).
   (ii) \( \text{NP} \cup \text{coNP} \) is contained in \text{DP}.

(c) (3 points) What is the difference between \text{NC}^i \text{ and } \text{AC}^i?\)

(d) (11 points) Recall that \text{EXP} is the union, over all polynomials \( p \), of the classes \( \text{DTIME}(2^{p(n)}) \) and that \text{NEXP} is the union, over all polynomials \( p \), of the classes \( \text{NTIME}(2^{p(n)}) \). Prove that, if all unary languages in \text{NP} are in \text{P}, then \text{EXP} = \text{NEXP}.