CPSC 468/568 Exam
March 1, 2007

Answer five of the following six questions. If you answer all six, the first five of your answers will be graded, and the sixth will be ignored. Please remember to write your name, CPSC 468/568, and today's date on the covers of all blue books you submit.

Question 1

(a) (2 points) Let A be the set of pairs (α, x) such that Turing Machine M_α halts in at most $|x|^3$ steps on input x. Is A undecidable? Give a yes-or-no answer and justify it in one sentence.

(b) (3 points) Define the term *oblivious Turing Machine*.

(c) (15 points) Construct an oracle O such that NP^O is not equal to $coNP^O$.

Question 2

For purposes of this question, use the fact that the IND-SET (independent set) problem is NP-complete. Prove that the following three problems are also NP-complete.

(a) (5 points) VC (vertex cover): the set of all pairs (G, k), where G = (V, E) is a graph and there is a subset V' of V such that $|V'| \leq k$ and, for every edge (u, v) in E, at least one of u or v is in V'.

(b) (5 points) SUBGRAPH-ISO (subgraph isomorphism): the set of all pairs (G, H) such that G contains a subgraph that is isomorphic to H. An isomorphism f from graph G_1 to graph G_2 is a bijective mapping from $V(G_1)$ to $V(G_2)$ such that, for all v_1 and v_2 in $V(G_1)$, $(v_1, v_2) \in E(G_1)$ if and only if $(f(v_1), f(v_2)) \in E(G_2)$.

(c) (10 points) DOMINATING SET: the set of all pairs (G, k) such that there is a subset V' of V(G) such that $|V'| \leq k$ and every vertex v in $V(G) - V'$ is adjacent to at least one member of V'.

Question 3

(a) (7 points) Define *implicitly logspace-computable reductions*. Explain why they are needed in the study of nondeterministic logspace.

(b) (7 points) Define *read-once certificates*. Explain why they are needed in the study of nondeterministic logspace.

(c) (6 points) Prove that, for every space-constructible $s(n) \geq \log n$, NSPACE(s(n)) and coNSPACE(s(n)) are contained in $DTIME(2^{O(s(n))})$.

Question 4

State whether each of the following claims is true, false, or unknown. If you answer true or false, give a very brief justification.

(a) (4 points) NP = P/poly

(b) (4 points) PSPACE = NPSPACE

(c) (4 points) DSPACE(n) = NSPACE(n)

(d) (4 points) NSPACE(n) = coNSPACE(n)

(e) (4 points) TQBF is PH-complete.

Question 5

The PSPACE-complete language GEOGRAPHY is defined as follows. An instance consists of a directed graph G = (V, E) and a designated start node s ε V. Player I moves first by choosing node s; then player II moves by choosing a node s' ≠ s such that (s, s') ε E. More generally, after m moves have been made, exactly m nodes have been chosen, and one of the two players has chosen node u in the m^{th} move; the $(m+1)^{st}$ move is then made by the other player, who must choose a node v such that (u, v) ε E, and v has not already been chosen in one of the first m moves. When a player is unable to move (because no such node v exists), he loses. The instance (G, s) is a yes-instance of GEOGRAPHY if and only if player I has a winning strategy.

(a) (3 points) Construct a yes-instance of GEOGRAPHY.

(b) (3 points) Construct a no-instance of GEOGRAPHY.

(c) (14 points) Prove that GEOGRAPHY is in PSPACE.

Question 6

(a) (3 points) Define the complexity class DP.

(b) (3 points) Which of the following relationships is known?
(i) DP is contained in NP ∪ coNP.
(ii) NP ∪ coNP is contained in DP.

(c) (3 points) What is the difference between NC^i and AC^i?

(d) (11 points) Recall that EXP is the union, over all polynomials p, of the classes $DTIME(2^{p(n)})$ and that NEXP is the union, over all polynomials p, of the classes $NTIME(2^{p(n)})$. Prove that, if all unary languages in NP are in P, then EXP = NEXP.