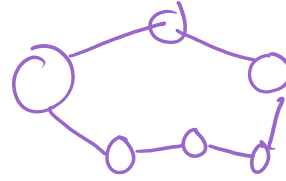
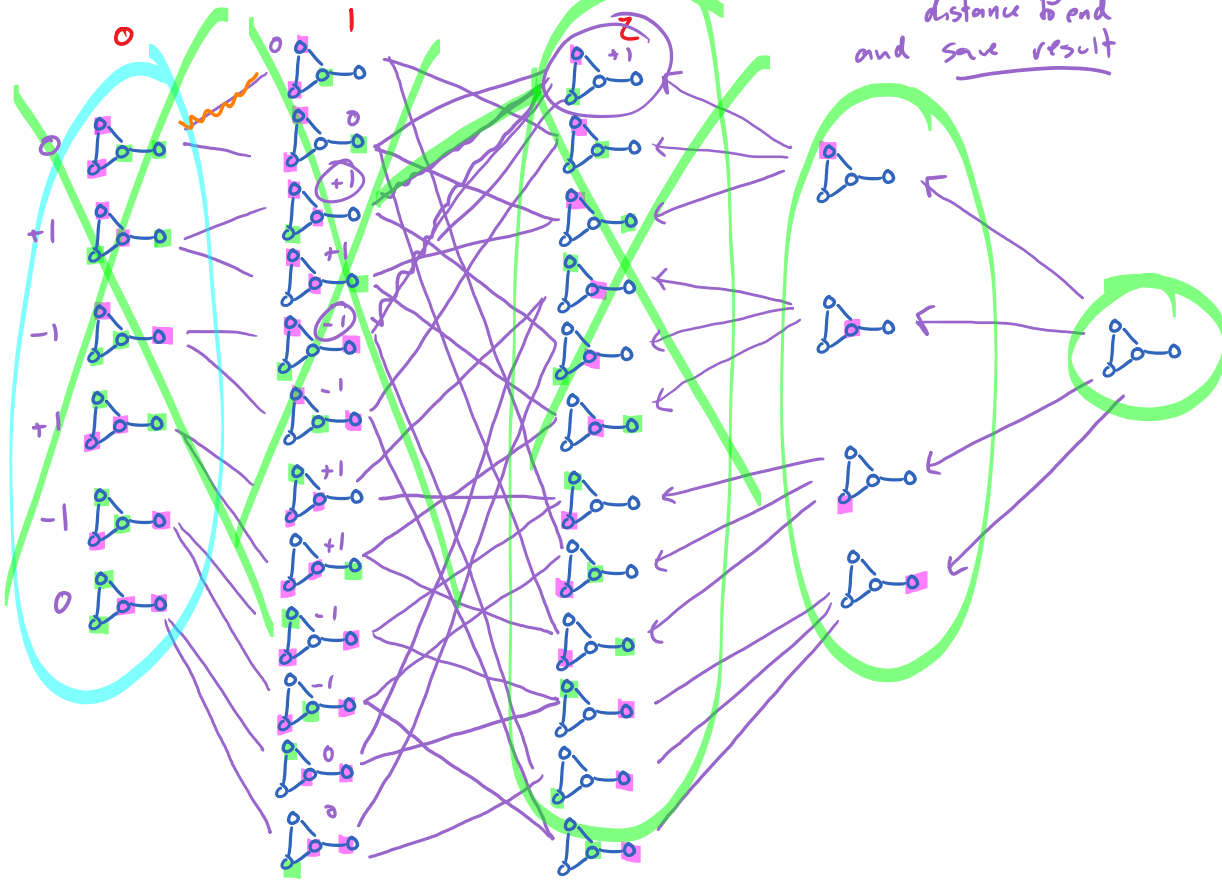


Dynamic Programming

Order positions by maximum distance to end.

Determine winner of distance 0 positions (terminal) by referring to rules

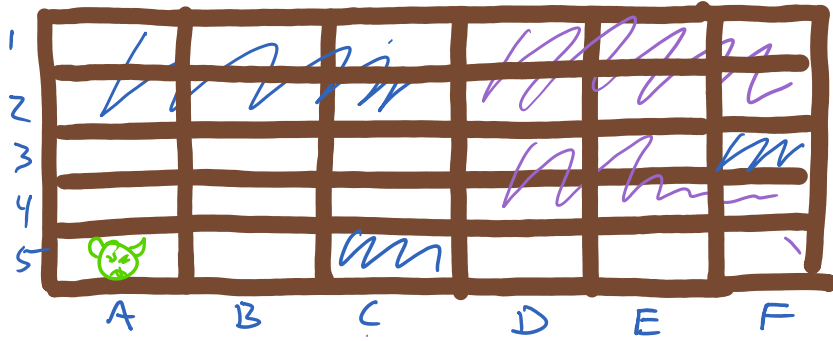
Use recursive formula to determine value of other positions in order of increasing distance to end and save result



Play on $m \times n$ grid. Take turns selecting remaining cell, remove all above and to right

Last move loses.

misere
impartial



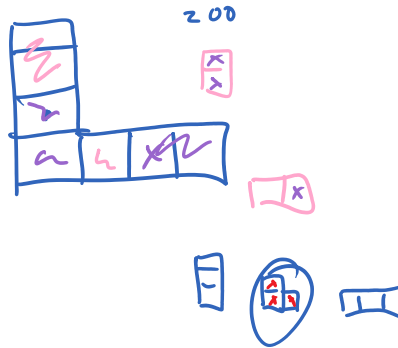
N: next player has winning move

P: prev. player has winning strategy

outcome-class(p)

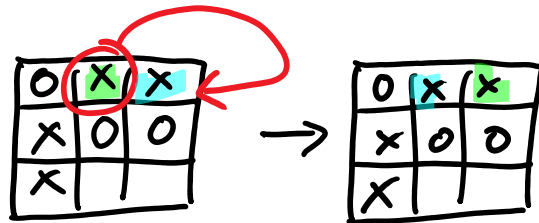
if p is end of game
return value according to rules
else
S ← positions reachable in 1 move from p
if S contains a P position
return N
else
return P

- 000
- 100
- 110
- 200
- 111
- 210
- 211
- 220
- 221
- 222



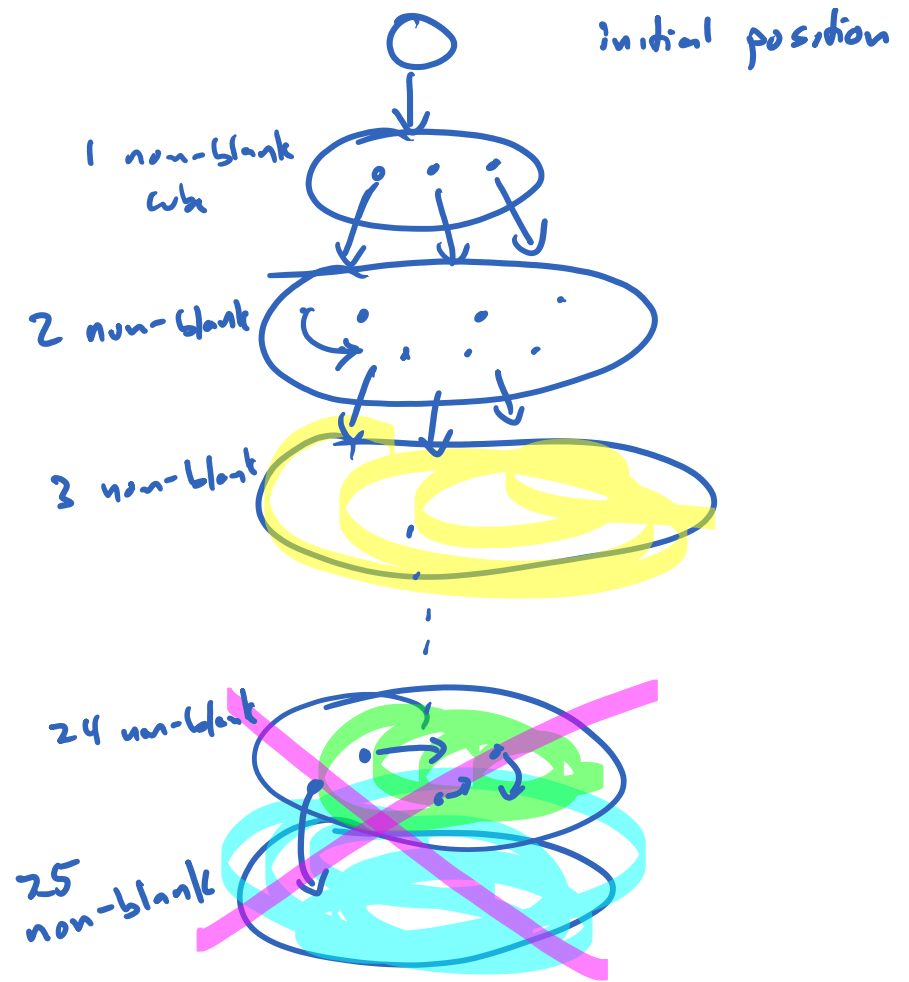
	X	O
	O	X

	X	O
	O	X



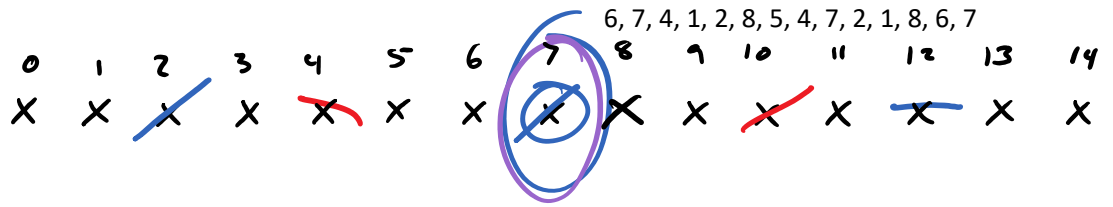
Value Iteration

mark all positions as draws
 mark all terminal positions according to rules
 repeat
 for all positions P marked as draws
 if some successor is win for current player @ P
 mark P as win for current player @ P
 else if all successors marked as losses
 mark P as win for other player
 until no new positions marked



Kayles

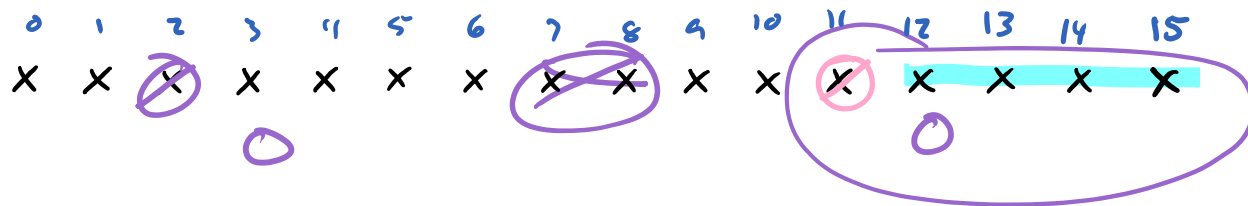
0, 1, 2, 3, 1, 4, 3, 2, 1, 4, 2, 6, 4, 1, 2, 7, 1, 4, 3, 2, 1, 4,
6, 7, 4, 1, 2, 8, 5, 4, 7, 2, 1, 8, 6, 7



Start with row of n pins

On each turn, take 1 or 2 adjacent pins

If no possible moves, you lose



Game Positions

Game position = set of positions one can move to

In traditional 1-row Nim

0 stones

$$\begin{aligned} \underline{U} &= \{ \} = \emptyset \\ \underline{0} &= \{ \underline{U} \} \\ \underline{00} &= \{ \underline{U}, \underline{0} \} \\ \underline{000} &= \{ \underline{U}, \underline{0}, \underline{00} \} \\ 0000 &= \\ &\vdots \end{aligned}$$