Blotto

Tuesday, October 11, 2022 11:36 AM

I 3 3 3 3 3 3

1 2 3 4 5 6 — 1+2+3=6 — 18 units

II wins

II 0 0 0 6 6 6 — 4+5+6=15

0 1 0 7 10 0 — 1+4+5=10

1 2 3 4 5 6 — 1+4+5=1

1 1 3 6 6 1 — 1+1+3+6=11

3 1 1 2 4 7 — 11

1 1 3 1 6 6

I $a_1 \; a_2 \; \dots \; a_6$

$b_1 \; b_2 \; \dots \; b_6$

$$\sum_{i=1}^{6} v_i \cdot \underbrace{\frac{a_i^2}{a_i^2+b_i^2}}_{P\ \text{I winning}}$$

maximize score, battlefield winners probabilistic

1 2 3

0 | 0 | 000 | 0 | 000000 | 000000

string of 18 0's 5 1's

#units #battlefields - 1

$$\binom{23}{5} = \binom{23}{18}$$

$$\binom{\#\text{units} + \#\text{battlefields} - 1}{\#\text{units}}$$

draw

1800000

180000 . . . 00 . . . 18

010710 0

$$\begin{pmatrix} \frac{1}{2} & & & \\ 0 & & & \\ 0 & & 1 & \\ 0 & & & \\ 0 & & & \\ 0 & & & \\ 0 & & & \\ 0 & & & \frac{1}{2} \end{pmatrix}$$

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-- verify : read X, determine if (X,X) is equil.

-- find : find X s.t. (X,X) is equil.

Linear Programming

	F	C	S
F	0.30	0.25	0.20
C	0.26	0.33	0.28
S	0.28	0.30	0.33

find $\underline{x} = (x_1 \ x_2 \ x_3)$ that satisfies

$E(x, 1) = 0.30x_1 + 0.26 \cdot x_2 + 0.28 \cdot x_3 \geq v$

$E(x, 2) = 0.25x_1 + 0.33x_2 + 0.30x_3 \geq v$

$E(x, 3) = 0.20x_1 + 0.28x_2 + 0.33x_3 \geq v$

$x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

and maximizes v

linear programming (scipy.linprog)

divide by v to eliminate v

(after ensuring that $v > 0$ [by adding a c to A to make all entries > 0])

$-0.30\frac{x_1}{v} - 0.26\frac{x_2}{v} - 0.28\frac{x_3}{v} \leq -1$

$-0.25\frac{x_1}{v} - 0.33\frac{x_2}{v} - 0.30\frac{x_3}{v} \leq -1$

$-0.20\frac{x_1}{v} - 0.28\frac{x_2}{v} - 0.33\frac{x_3}{v} \leq -1$

$\boxed{\text{let } p_i = \frac{x_i}{v}}$

maximize v

⇓

$\boxed{\text{minimize } p_1 + p_2 + p_3} = \frac{x_1}{v} + \frac{x_2}{v} + \frac{x_3}{v} = \frac{(x_1 + x_2 + x_3)}{v} = \frac{1}{v}$

c

(1, 1, 1)

$-0.30p_1 - 0.26p_2 - 0.28p_3 \leq -1$

$-0.25p_1 - 0.33p_2 - 0.30p_3 \leq -1$

$-0.20p_1 - 0.28p_2 - 0.33p_3 \leq -1$

b-ub

al

and $0 \leq p_i = \frac{x_i}{v} \leq \frac{1}{v} \leq 5$

since $x_i \leq 1$ because $v \geq 0.20$

lowest value in payoff matrix

bounds (0, 5)

result.x result.fun

LP returns (p_1, p_2, p_3) and $p_1 + p_2 + p_3 = \frac{1}{v}$

recover x_i with $x_i = p_i \cdot v = \frac{p_i}{p_1 + p_2 + p_3}$

$p_i = \frac{x_i}{v}$

$x_i = v \cdot p_i$

II

minimize v subject to

$E(1, y) = 0.30y_1 + 0.25y_2 + 0.20y_3 \leq v$

$E(2, y) = 0.26y_1 + 0.33y_2 + 0.28y_3 \leq v$

$E(3, y) = 0.28y_1 + 0.30y_2 + 0.33y_2 \leq v$

$E(2,Y) = 0.26y_1 + 0.33y_2 + 0.28y_3 \le v$

$E(3,Y) = 0.28y_2 + 0.30y_2 + 0.33y_3 \le v$

$\Downarrow$ divide by v, replace $\frac{y_i}{v}$ with q_i

find q_1, q_2, q_3 to minimize $\underline{-q_1 - q_2 - q_3} = -\frac{1}{v}$ $\left(\begin{array}{c} \min -\frac{1}{v} \\ \equiv \\ \max \frac{1}{v} \\ \equiv \\ \min v \end{array}\right)$

$c = (-1, -1, \ldots -1)$

$$a1 = A \quad \begin{array}{l} 0.30q_1 + 0.25q_2 + 0.20q_3 \le 1 \\ 0.26q_1 + 0.33q_2 + 0.28q_3 \le 1 \\ 0.28q_1 + 0.30q_2 + 0.33q_3 \le 1 \end{array}$$

$b\_ub = (1, \ldots, 1)$

$\underline{0} \le q_i \le \underline{5}$

bounds

LP returns q_i and $-q_1 - q_2 - q_3$ $\qquad v = \frac{1}{q_1 + q_2 + q_3}$

recover y_i with $y_i = q_i \cdot v = \frac{q_i}{q_1 + q_2 + q_3}$

$\sum$

Non-constant sum

X, Y is equilibrium if and only if $E_I(i, Y) \le E_I(X, Y)$ for all i
$E_{II}(X, j) \le E_{II}(X, Y)$ for all j

	F	O
F	4, 1	0, 0
O	0, 0	1, 3

$X = \left(\frac{3}{4} \quad \frac{1}{4}\right)$ $Y = \left(\frac{1}{5} \quad \frac{4}{5}\right)$

value of game for I

value of game for II

Non linear programming : maximize $\underbrace{E_I(X, Y)}_{\sum_{i=1}^{n}\sum_{j=1}^{m} x_i \cdot y_j \cdot a_{ij}} + E_{II}(X, Y) - p - q$

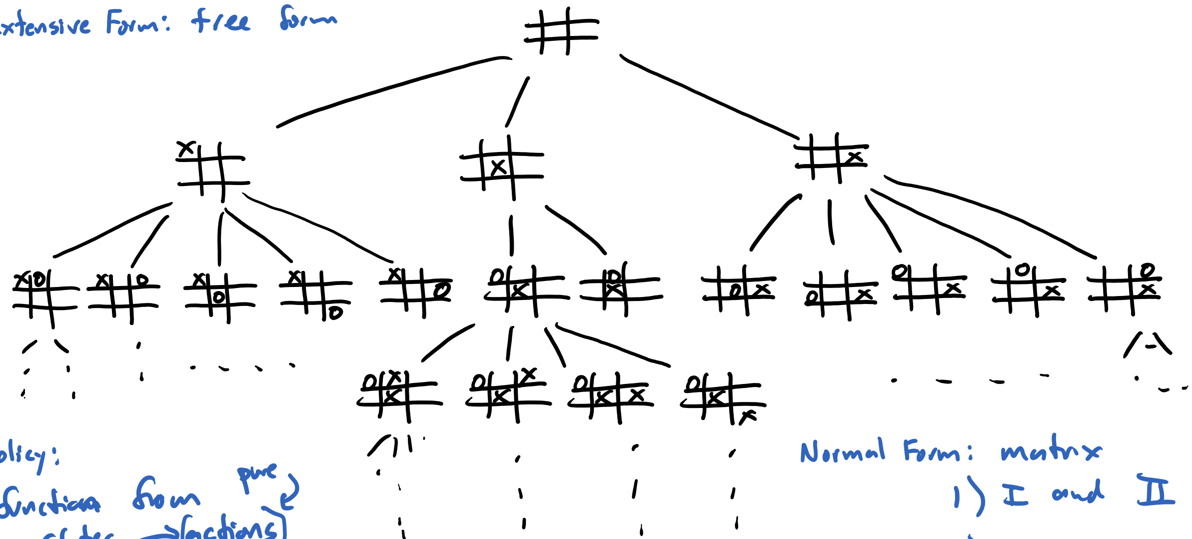
subject to

non linear!

$$A = \begin{pmatrix} \frac{3}{2} & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & \frac{5}{4} \\ 1 & -1 & 0 \\ 0 & 1 & \frac{5}{4} \end{pmatrix}$$

Extensive Form vs Normal Form

Extensive Form: free form



Policy:

functions from states → {actions} pure

mixed: prob dist over actions

Normal Form: matrix

1) I and II pick policies

2) play game according to policies

$$\begin{pmatrix} -1 & & \\ & 0 & 1 \\ & & \end{pmatrix}$$

an action ≈ 6 options

chess 6 ≈ 40

state 0 — — — . . . . . . — — 10^{70}

\# pure strategies = $40^{(10^{70})}$

One-Card Poker

Each player antes 1 unit

Deal one card to each player

P1 bets 1 or 0

bet "check"

P2: call (add 1 to pot) | fold | bet | check

P1: call | fold

4 card deck A, K, Q, J of clubs

high … low

P1 strategy:

1	1	0	1	X	X	0	X
A	K	Q	J	A01	K01	Q01	J01

P1's card — betting history

P2 strategy:

1	1	0	0	1	1	0	0
A0	K0	Q0	J0	A1	K1	Q1	J1

P1 checked — P1 bet

P1	P2	
A	K	+2
A	Q	+1
A	J	+1
K	A	−2
K	Q	+1
K	J	+1
Q	A	−1
Q	K	−1
Q	J	+1
J	A	−2
J	K	−2
J	Q	+1

expected payoff for I $= \frac{1}{12} \cdot 2 + \frac{1}{12} \cdot 1 + \cdots + \frac{1}{12} \cdot 1$

$= 0$

256 cols

11001100

≤ 256 rows

1101XX0X

$$\begin{pmatrix} & & \\ & 0 & \\ & & \end{pmatrix}$$