YOUR NAME PLEASE: DA 4/26/15

SOLUTIONS

Computer Science 201b
Practice Final Exam
Spring 2015
2.5 hour exam + .5 hour of writing up

Closed book and closed notes. Show ALL work you want graded on the test itself.

For problems that do not ask you to justify the answer, an answer alone is sufficient. However, if the answer is wrong and no derivation or supporting reasoning is given, there will be no partial credit.

GOOD LUCK!

problem	points	actual
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
total	120	

1.(a) (6 points)
Write a Racket procedure (insert x lst) that takes as input a number x and a list lst of numbers in non-decreasing order, and returns a list of numbers in non-decreasing order that includes x and all the elements of lst. There may be duplicates in the lists.

Examples:

```
(insert 7 '(1 4 5 10 12)) => '(1 4 5 7 10 12)
(insert 3 '()) => '(3)
(insert 2 '(1 2 4)) => '(1 2 2 4)
(insert 2 '(1 2 2 4)) => '(1 2 2 2 4)
```

```
(define (insert x lst)
  (cond
    [(null? lst) (list x)]
    [(<= x (first lst))
      (cons x lst)]
    [else
      (cons (first lst)
            (insert x (rest lst)))]))
```

1.(b) (4 points)
Assuming that the insert procedure works as in part (a), use it to write a procedure (isort lst) that takes a list lst of numbers and returns the list of numbers sorted into non-decreasing order. There may be duplicates.

Examples:

```
(isort '(6 3 10 1 5)) => '(1 3 5 6 10)
(isort '()) => '()
(isort '(4 2 2 1 2)) => '(1 2 2 2 4)
```

```
(define (isort lst)
  (if (null? lst)
      '()
      (insert (first lst)
              (isort (rest lst)))))
```

1.(c) (2 points)
Give an example of a number x and a list lst of n elements in non-decreasing order that make your insert procedure from part (a) run for

(i) the shortest time (best case):

x = 0   lst = (1 2 ... n)

(ii) the longest time (worst case):

x = n+1   lst = (1 2 ... n)

2. An and/or expression is recursively defined. The base case is a Boolean value or a symbol. The recursive case is a list containing three elements:
1. the left operand, which is an and/or expression,
2. the operation symbol, '+ or '*,
3. the right operand, which is an and/or expression.

Examples: 'x, #t, '(#t + #f), '(((x + y) * (u + v)) + (a + b))

2.(a) (3 points)
If exp is an and/or expression that is not a symbol or a Boolean value, what Racket expressions will give the following parts of exp (the first one is done):

(i) the left operand: (first exp)

(ii) the operation symbol: (second exp)

(iii) the right operand: (third exp)

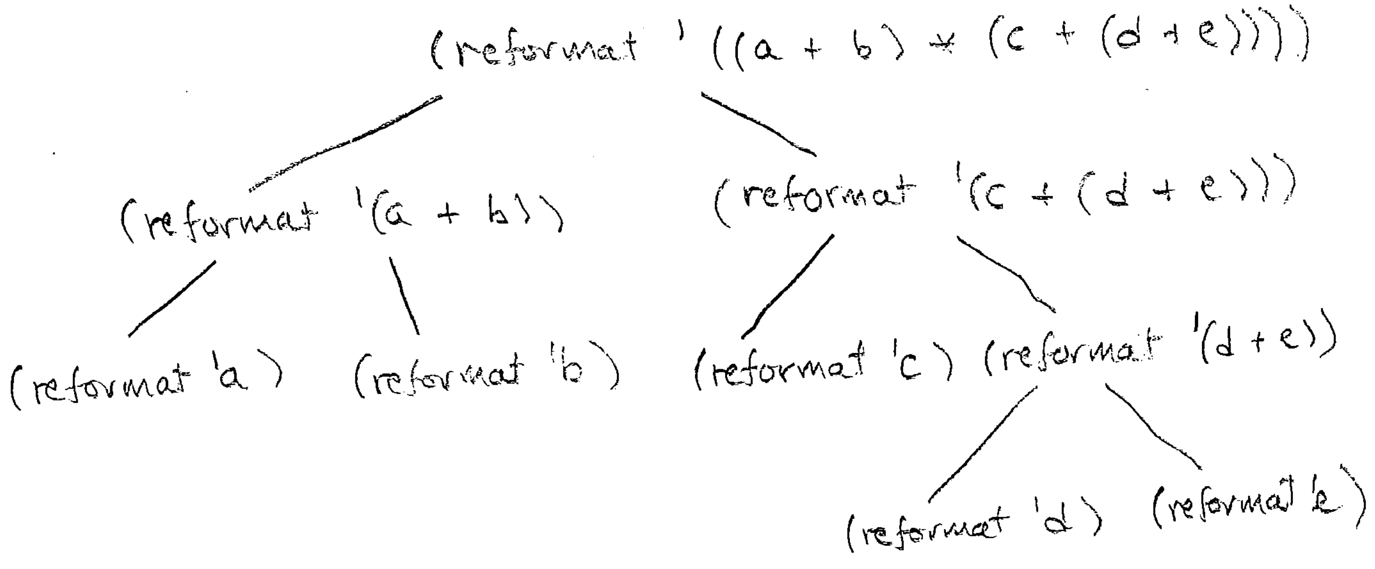
2.(b) (6 points)
Write a Racket procedure (reformat exp) that takes an and/or expression exp and reformats it so that the operation symbol comes first in the list, followed by the left operand and then the right operand.
Examples:

```
(reformat #t) => #t
(reformat 'hi) => 'hi
(reformat '(x + y)) => '(+ x y)
(reformat '(((x + y) * (u + v)) + (a + b))) => '(+ (* (+ x y) (+ u v)) (+ a b))
```

```
(define (reformat exp)
  (if (not (list? exp))
      exp
      (list
        (second exp)
        (reformat (first exp))
        (reformat (third exp)))))
```

2.(b) (more space)

2.(c) (3 points)
Draw the tree of recursive calls (and no return values) for the procedure call:
(reformat '((a + b) * (c + (d + e)))).



(will depend on procedure in 2(b))

3. Recall that the TC-201 assembly-language instructions are:

```
    halt, load address, store address, add address, sub address,
    input, output, jump address, skipzero, skippos, skiperr,
    loadi address, storei address
```

and the directive: data number, which reserves one memory location and stores the number in it.

3.(a) (10 points)
Write a TC-201 program in assembly language that reads in a number, n and then prints out the first n odd positive integers, in increasing order, and then halts. You may assume that n is between 1 and 5000, inclusive. You may use symbolic addresses. An example of the behavior of the program:

```
input = 4
output = 1
output = 3
output = 5
output = 7
```

```
      input
      store n
      load one
      store odd
loop: load odd
      output
      add two
      store odd
      load n
      sub one
      skippos
      halt
      store n
      jump loop
n:    data 0
odd:  data 0
one:  data 1
two:  data 2
```

3.(b) (1 point)
What is the largest value of n for which we could expect to have a TC-201 program output the first n odd positive integers in increasing order. Exponential notation is fine. Please justify your answer.

$n = 2^{14}$ (= 16384)

because the n^{th} positive integer is $2n-1$ and $2 \cdot 2^{14} - 1 = 2^{15} - 1$, which is the largest integer representable in the TC-201.

3.(c) (1 point)
Could there exist a Racket program that takes as input a configuration config of the TC-201 computer and correctly decides whether starting in config the machine would eventually reach a configuration with the run-flag equal to 0? Assume that the user types 0 in response to every input instruction. Please justify your answer.

Yes, such a program could be written. It would simulate the TC-201 from config, keeping a list of all the configurations reached.

If a configuration with run-flag = 0 is reached, return YES.

If a configuration is reached that is equal to a previously-reached configuration (with run-flag = 1), then the TC-201 program will loop forever, so return NO.

Because the total number of configurations is FINITE, one or the other of these must eventually happen.

4.
You are to design a combinational circuit with
four inputs: a1, a0, b1, b0, and two outputs: c1, c0.
a1 and a0 are interpreted as a 2-bit binary number A,
b1 and b0 are interpreted as a 2-bit binary number B, and
c1 and c0 are interpreted as a 2-bit binary number C.
The value of C should be the minimum of A and B.

For example, if a1 = 1 and a0 = 0 then A = 2 because 10
in binary is 2.

4.(a) (6 points)
Complete the rows of the truth table giving c1 and c0 as
a function of a1, a0, b1, b0:

a1	a0	b1	b0	c1	c0
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	1	1

Note that for the row shown, a1 and a0 are 0 and 1, so A = 1,
and b1 and b0 are 1 and 0, so B = 2. Thus, C = min(A,B) = 1,
so c1 = 0 and c0 = 1.

4.

4.(b) (2 points)

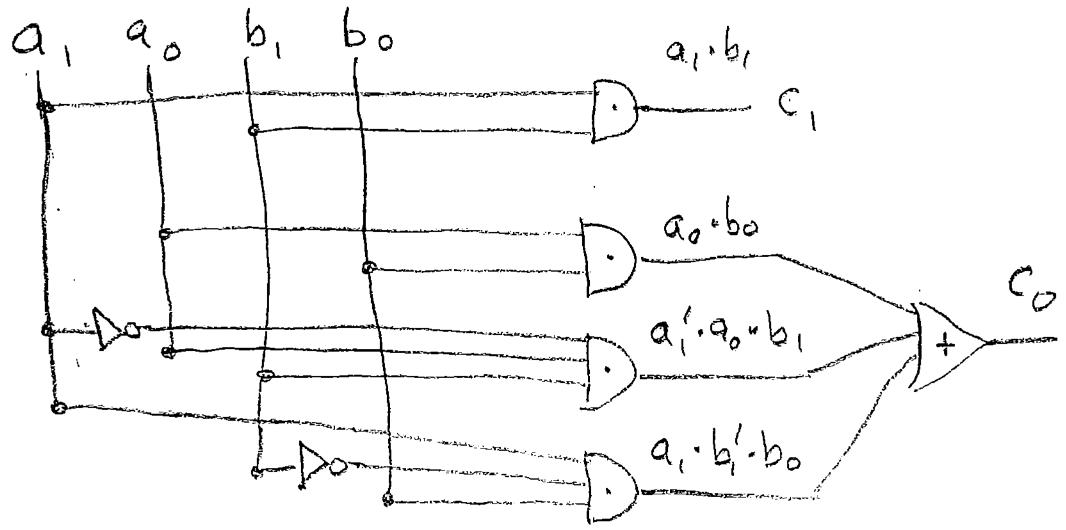
Give Boolean expressions for c1 and c0.

c1 = $a1 \cdot b1$

c0 = $a0 \cdot b0 + a_1' \cdot a_0 \cdot b_1 + a_1 \cdot b_1' \cdot b_0$

4.(c) (4 points)

Draw a combinational circuit for computing outputs c1 and c0 from inputs a1, a0, b1, b0. You may use AND and OR gates of any number of inputs, NOT gates of one input, and XOR gates of two inputs. You may draw gates as simple boxes; be sure to label your input and output wires and your gates (if you don't use the standard symbols.)

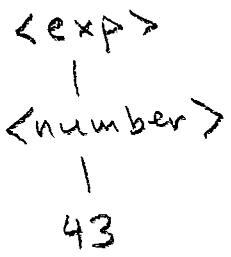


5. The following is a context-free grammar in BNF for a small subset of syntactically correct Racket expressions.

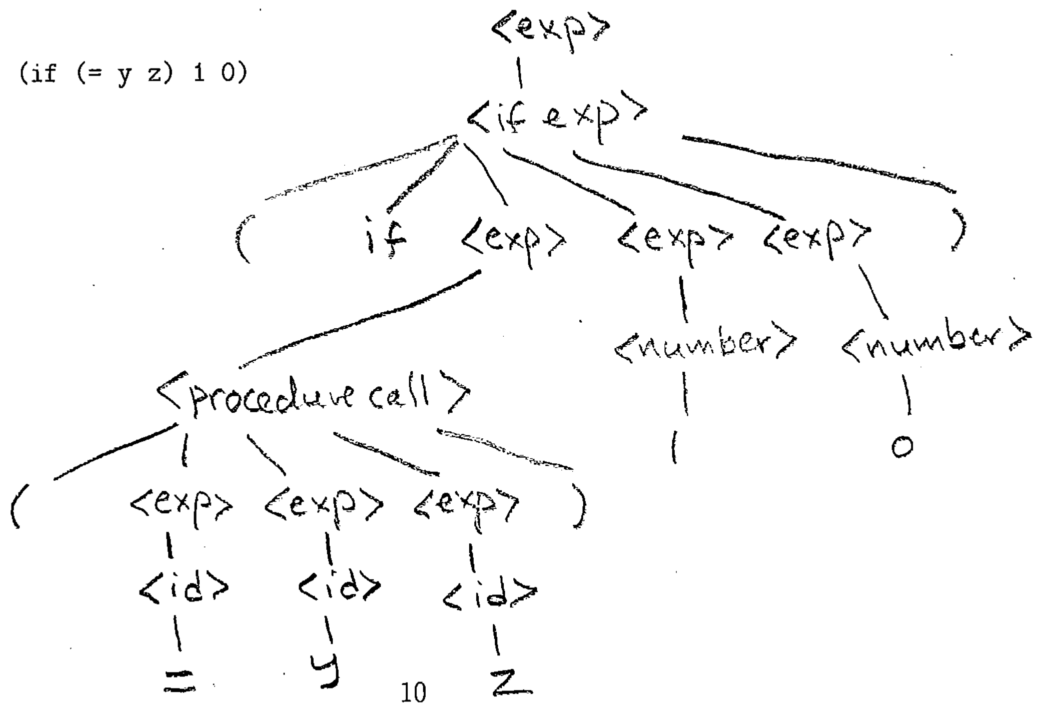
```
<exp> ::= <id> | <number> | <boolean> | <lambda exp>
        | <procedure call> | <if exp>

<id> ::= "x" | "y" | "z" | "+" | "*" | "="
<number> ::= "0" | "1" | "3" | "7" | "43"
<boolean> ::= "#t" | "#f"
<lambda exp> ::= "(" "lambda" <formals> <exp> ")"
<formals> ::= "(" <exp>* ")"
<procedure call> ::= "(" <exp> <exp>* ")"
<if exp> ::= "(" "if" <exp> <exp> <exp> ")"
```

5.(a) (3 points each)
For each expression below, draw a parse tree showing how it can be derived from <exp> using the rules above.

(i) 43

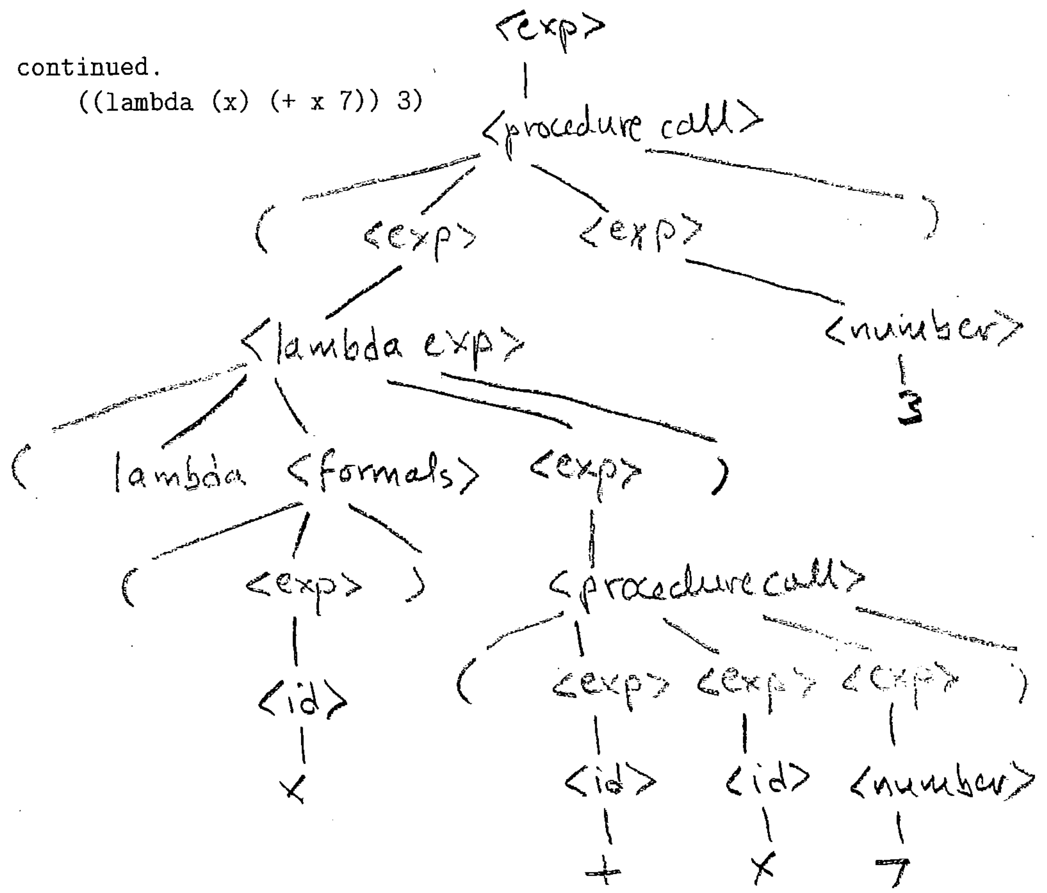


(ii) (if (= y z) 1 0)



5.(a) continued.

(iii) ((lambda (x) (+ x 7)) 3)



5.(b) (3 points)
If L is a context-free language and reverse(L) is the set of all reverses of strings in L, must reverse(L) also be a context-free language? Please briefly justify your answer.

Yes.
Given a context free grammar for L, if we reverse the right hand side of every rule, we get a context free grammar for reverse(L).

6. Consider a Racket procedure (make-set name) that takes a symbol name as input and returns a Racket procedure that implements a set object with local storage which can process the following commands:

```
name             -- returns the name of the set
contains? value  -- returns #t if value is a member of the
                    set, or #f if value is not a member of the set
include value    -- changes the set so that value is a member of
                    the set, and returns the symbol 'ok
```

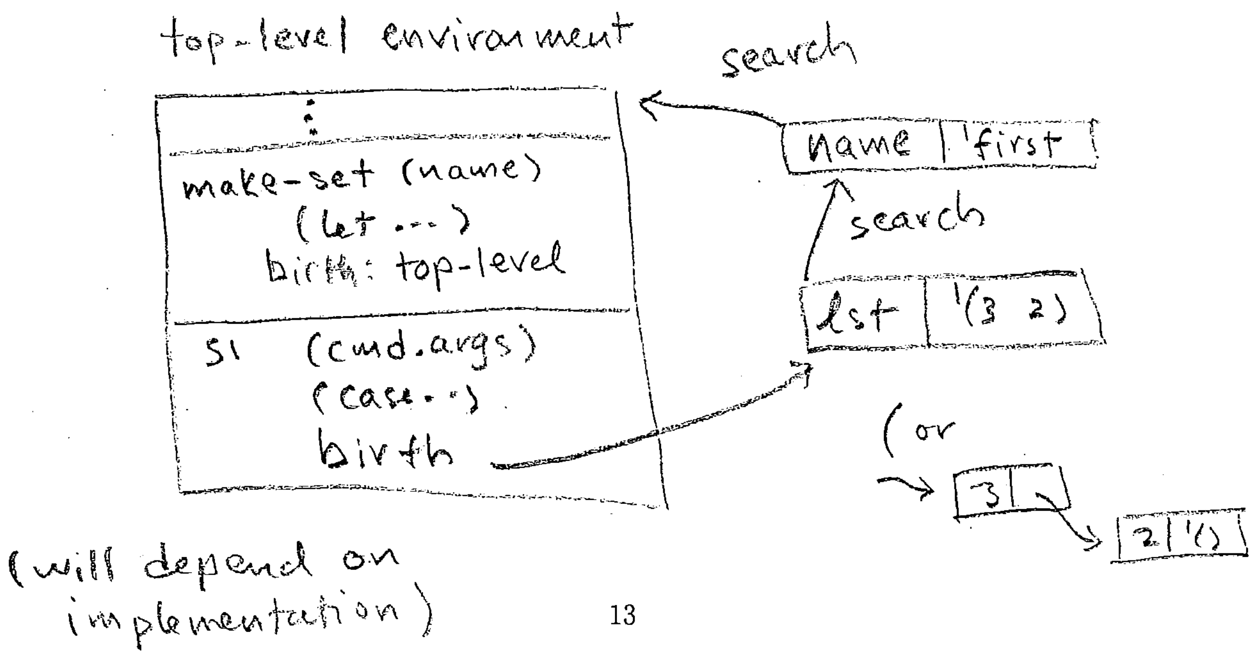
Initially the set has no members. Examples of using (make-set name):

```
> (define s1 (make-set 'first-set))
> (s1 'name)
 'first-set
> (s1 'contains? 2)
 #f
> (s1 'include 2)
 'ok
> (s1 'include 3)
 'ok
> (s1 'contains? 2)
 #t
> (define s2 (make-set 'second-set))
> (s2 'contains? 2)
 #f
> (s2 'contains? 'second-set)
 #f
> (s1 'contains? 3)
 #t
> (s2 'name)
 'second-set
>
```

6.(a) (9 points)
Write a Racket procedure to implement (make-set name).
No error checking is necessary.

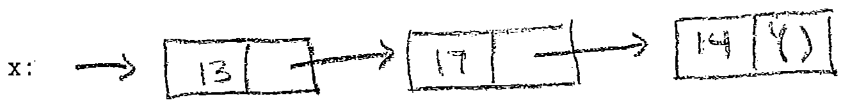
```
(define (make-set name)
  (let ((lst '()))
    (lambda (cmd . args)
      (case cmd
        [(name) name]
        [(contains?)
          (if (member (first args)) lst)
              #t
              #f) ]
        [(include)
          (set! lst (cons (first args) lst))
          'ok ]))))
```

6.(b) (3 points)
Describe or draw the accessible environments immediately after the completion of the (s1 'include 3) command above. Include and label the search pointer of each environment and the birth pointers of the procedures make-set and s1.



7.(a) (2 points)
Draw the box and pointer representation of the list x constructed as follows.

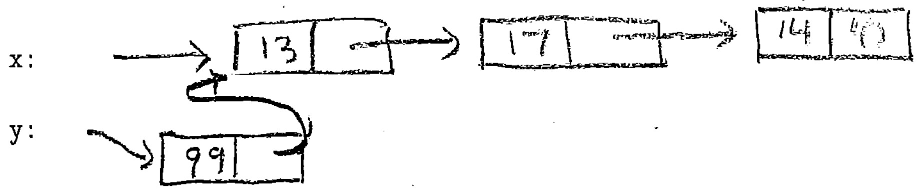
```
> (define x (cons 13 (cons 17 (cons 14 '()))))
> x
'(13 17 14)
>
```



7.(b) (2 points)
Suppose x is as in 7(a) and we define y as follows

```
> (define y (cons 99 x))
> y
'(99 13 17 14)
>
```

Draw a new box and pointer diagram showing the resulting structure of x and y.



7.(c) (2 points)
Explain why we can consider the Racket procedures rest (or cdr) and cons to be constant time, that is, O(1).

Because
rest involves a constant number of assembly-language instructions to access the value in the right half of a cons cell
cons involves a constant number of assembly-language instructions to allocate a cons cell and deposit the values of its arguments in the left and right halves of the cons cell.

7.(d) (3 points)
Consider the following Racket procedure.

```
(define (rev lst rlst)
  (if (null? lst)
      rlst
      (rev (rest lst) (cons (first lst) rlst))))
```

Draw the tree of recursive calls (and no returns) for the procedure call (rev '(a b c) '()).

```
(rev '(a b c) '())
   |
(rev '(b c) '(a))
   |
(rev '(c) '(b a))
   |
(rev '() '(c b a))
```

7.(e) (3 points)
For the procedure in 7(d) suppose we call (rev lst '()) where the list lst contains n numbers. Give a "Big Theta" bound in terms of n for the running time of this procedure call, and explain why it is correct.

Time is $\Theta(n)$ because each call does a null? test ($\Theta(1)$) and if the list is not null, a recursive call ($\Theta(1)$) with the rest of lst ($\Theta(1)$) and (cons (first lst) rlst) (also $\Theta(1)$). After n recursive calls, the value of lst will be '(), and the value of rlst is returned ($\Theta(1)$). Thus, the total time is $\Theta(n)$.

8. For each of the following pairs consisting of a string and a regular expression, determine whether the string is in the language of the regular expression or not, answering YES or NO. Recall that | stands for "or" and * for "Kleene star", that is, zero or more repetitions.
In each case the alphabet is {a,b,c}.

(2 points each)

8.(a) aaccc (a)*(b)*(c)*

YES

8.(b) abaacbcc (a|b)*(b|c)*

YES

8.(c) abcbc ((ab | ba)* | (bbc | cbc)*)

NO

8.(d) bccbacba (bc | bcc)(bac | cba)(cba | aa)

YES

```
8.(e)  accbcc        ((a | b)c(c)*)*
```

YES

```
8.(f)  cabacbbbb     (ca | ba | aa | cb | bb | ab)*
```

NO

9. Answer each question briefly. (2 points each)

9.(a) What is the main thing a compiler does?

Translate a program in a higher-level language into an equivalent assembly-language (or machine-language) program.

9.(b) Give an example of a set of strings that is not regular.

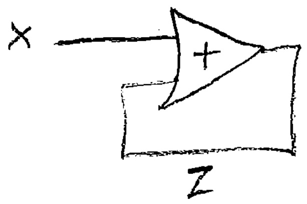
$L = \{a^n b^n \mid n \geq 0\}$

The set of strings consisting of n a's followed by n b's. (or many others)

9.(c) What is memoization, and when is it useful?

Memoization saves argument/result pairs during a computation and looks up the results instead of recomputing them. It is useful to avoid repeatedly recomputing results during a computation.

9.(d) Give a small example of a sequential (not combinational) circuit.



(or any other circuit with a "loop")

9.(e) We proved that there can be no program to solve the Halting Problem. State the Halting Problem for Racket programs.

There is no procedure (halts? proc exp) that returns #t if (proc exp) halts and #f if (proc exp) doesn't halt.

9.(f) Is the following procedure tail recursive? Why or why not?

```
(define (proc lst n)
  (if (null? lst)
      n
      (proc (cdr lst) (+ n 1))))
```

Yes.
The only recursive call does not modify the value of (proc (cdr lst) (+ n 1)) before returning it.

10. A *rewriting system* R consists of a finite alphabet of symbols and a finite set of rules. Each rule is of the form s1 -> s2, where s1 and s2 are strings of symbols. The lefthand side of the rule is s1 and the righthand side of the rule is s2.

A *derivation step* is a pair of strings (t,u) such that for some rule s1 -> s2, u is obtained from t by replacing one occurrence of s1 as a substring of t by s2. A *completed derivation* is a finite sequence of strings (t1, t2, ..., tn) such that
(1) each consecutive pair forms a derivation step, and
(2) no rule applies to the last string, tn.
In this case, we say t1 *may yield* tn.

As an example, consider a rewriting system with alphabet {x,y} and the single rule yx -> xy. For this system, we have a completed derivation:

yyxyx, yxyyx, yxyxy, yxxyy, xyxyy, xxyyy.

Note that no rule applies to the last string. Thus, yyxyx may yield xxyyy.

10.(a) (3 points)
In the example one-rule system above, if a string s may yield a string t, describe how are s and t related.

t is equal to a rearrangement of s so that all the x's are before all the y's.

10.(b) (6 points)
For the alphabet {0,1} construct rules for a rewriting system such that for every non-empty string s, s may yield exactly one of 0 or 1, and s may yield 1 if and only if the number of 1's in s is odd.
(You'll need more than one rule.)

Examples of the behavior of this system:

```
101110  may yield 0, but not 1.
1101101 may yield 1, but not 0.
0000    may yield 0, but not 1.
```

$00 \to 0$
$01 \to 1$
$10 \to 1$
$11 \to 0$

10.(c) (3 points)
We define a decision problem as follows. The input is a rewriting system R and a string s. (We assume the alphabet of R contains 1.)
The output is "yes" if s may yield 1 in the system R, and "no" otherwise.

Is this a computable problem or not? Please briefly justify your answer.

No.
We could represent Turing machine configurations as strings, and Turing machine instructions as rules in such a way that steps of the computation would be simulated by applications of the rules. We could make sure the rules would reduce a configuration to 1 if and only if it was a halted configuration. Then the question would solve the Halting Problem for Turing machines, which we know is not computable.