YOUR NAME PLEASE: *** SOLUTIONS ***

Computer Science 201b
SAMPLE Exam 1 SOLUTIONS
February 15, 2015

Closed book and closed notes. No electronic devices. Show ALL work you want graded on the test itself.

For problems that do not ask you to justify the answer, an answer alone is sufficient. However, if the answer is wrong and no derivation or supporting reasoning is given, there will be no partial credit.

GOOD LUCK!

problem	points	actual
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
total	60	

1.(a) (5 points)
Define a Racket procedure (change item1 item2 lst)
that changes every top-level occurrence of item1 in lst to item2.

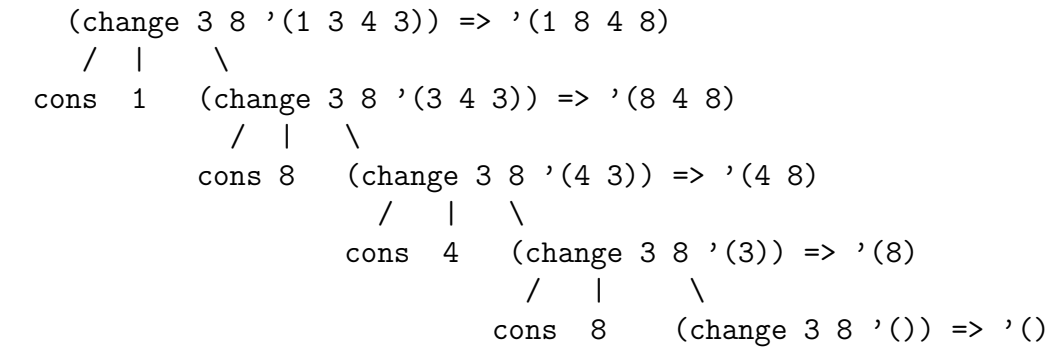
Use only the built-in Racket special forms and procedures:
define, lambda, null?, empty?, equal?, if, cond, car, first,
cdr, rest, cons, and quoted constant lists like '().

Do not write any auxiliary procedures for this problem.

Examples:
(change 3 4 '(4 5 3 4 4 3 0)) => '(4 5 4 4 4 4 0)
(change 'a 'b '(a l a s (a n d))) => '(b l b s (a n d))
(change '(0) 'x '(1 (0) 1 (1) (0))) => '(1 x 1 (1) x)

```
(define (change item1 item2 lst)
  (cond
    [(null? lst)
     '()]
    [(equal? item1 (first lst))
     (cons item2 (change item1 item2 (rest lst)))]
    [else
     (cons (first lst) (change item1 item2 (rest lst)))]))
```

1.(b) (5 points)
Draw a tree of recursive calls, with return values,
for your procedure when evaluating (change 3 8 '(1 3 4 3)).



2. Consider the boolean function f presented by the following truth table:

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

2. (a) (5 points)
Write down a Boolean expression
(using dot for AND, plus for OR and prime for NOT)
to represent f using the variables x, y, z.

Justify its correctness.

    x'*y'*z' + x*y'*z' + x*y*z'

(dots shown as * here)
via the sum-of-products algorithm.

2. (b) (5 points)
Give a Boolean expression (using AND, OR, NOT as in part (a))
corresponding to each of the following statements involving
the Boolean variables P, Q, and R.

You need not justify correctness.

(i) At least one of P, Q, R is 1.

  (P + Q + R)

(ii) At most one of P, Q, R is 1.

  P'*Q' + P'*R' + Q'*R'

(iii) R is 1 if P and Q are both 0, and R is 0 otherwise.

  (P+Q)*R' + P'*Q'*R

(iv) P, Q and R all have the same value.

  P*Q*R + P'*Q'*R'

(v) Exactly two of P, Q, R are 0.

  P*Q'*R' + P'*Q*R' + P'*Q'*R

3. Define an nb-list to be a list that is
empty or has as elements numbers, booleans, or nb-lists.
For example, the list '(#t 1 (2 ()) (#f #f))) is an nb-list.

3. (a) (6 points) Define a recursive Racket procedure
(numbers lst) that takes an nb-list lst and returns a list of all
the numbers that occur in lst, in the left-to-right
order in which they occur. Preserve duplicates.

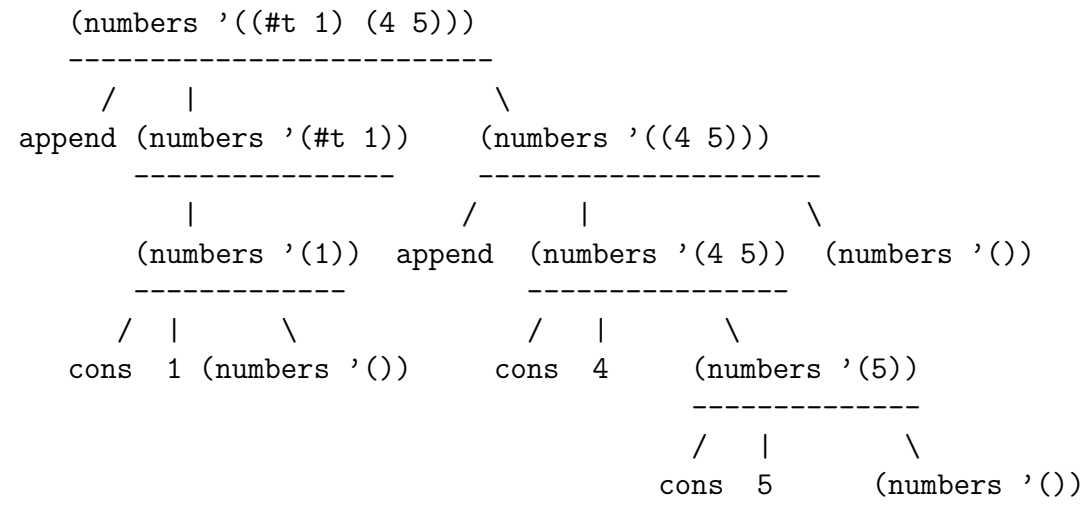
Use only the built-in Racket special forms and procedures:
define, lambda, null?, empty?, symbol?, boolean?, number?,
if, cond, car, first, cdr, rest, cons, append, and quoted
constant lists like '().

Do not write any auxiliary procedures for this problem.

Examples:
(numbers '(#f 1 (2 ()) ((1)))) => '(1 2 1)
(numbers '(#t 2 ((#f)) 26 #f (3 2))) => '(2 26 3 2)

```
(define (numbers lst)
  (cond
    [(null? lst)
     '()]
    [(number? (first lst))
     (cons (first lst) (numbers (rest lst)))]
    [(list? (first lst))
     (append (numbers (first lst))
             (numbers (rest lst)))]
    [else
     (numbers (rest lst))]))
```

3.(b) (4 points)
Draw a tree of recursive calls (no return values)
for your procedure numbers from part 3.(a)
when you evaluate (numbers '((#t 1) (4 5))).



4.(a) (8 points)
Construct a Turing machine to multiply
a unary number by 2. In the initial configuration the
tape has a contiguous sequence of m 1's, and all other
symbols are blank, the machine is in state q1 and the
head is on the leftmost 1. In the final configuration
the tape has a contiguous sequence of n 1's, and all
other symbols are blank, the machine is halted and
the head is on the leftmost 1.

Moreover, n = 2m.

For example, for input 111 the output should be 111111.
You may use additional tape symbols. You may assume m > 0.
You must give a clear explanation of how your machine works --
this is worth half the points on this problem.

The machine will repeatedly change the next 1 into an x
and write an x at the left end of the tape, until all the 1's
are converted to x's. Then it will rewrite all the x's as
1's and restore the head to the start of the sequence of 1's.

```
(q1, x, q1, x, R)      q1 runs right over x's
(q1, b, q3, b, L)      if it finds a blank, all 1's are converted, go to q3
(q1, 1, q2, x, L)      if it finds a 1, it converts it to x, and goes to q2
(q2, x, q2, x, L)      q2 runs left over x's until blank
(q2, b, q1, x, R)      and writes a new x there and goes back to q1
(q3, x, q3, 1, L)      q3 moves left over x's, changing them to 1's
(q3, b, q4, b, R)      until it finds a blank, when it goes to q4 to halt
```

4.(b) (2 points) An initial configuration for your machine is given below with the head on the leftmost of a sequence of of two 1's, the rest of the tape blank, in state q1.

Please show the next six configurations in order.
Number them 2 through 7.
(Note that b is the blank symbol.)

```
1)  b 1 1 b        5) x x x b
      ^                  ^
      q1                 q2

2)  b x 1 b        6) x x x b
    ^                  ^
    q2                 q2

3)  x x 1 b        7)  b x x x b
      ^                  ^
      q1                 q2

4)  x x 1 b
        ^
        q1
```

5. (10 points)

5.(a) Give a careful statement of the theorem from lecture
about the Halting Problem for Racket programs.

There is no procedure (halts? proc expr) that for any
Racket procedure proc and Racket expression expr will
always return #t if (proc expr) halts and #f if (proc expr)
does not halt.

5.(b) Give an example of the use of the cd command
in Linux, and explain briefly what it does.

```
> cd cs201
>
```

Changes the current directory (or folder) to the given one.

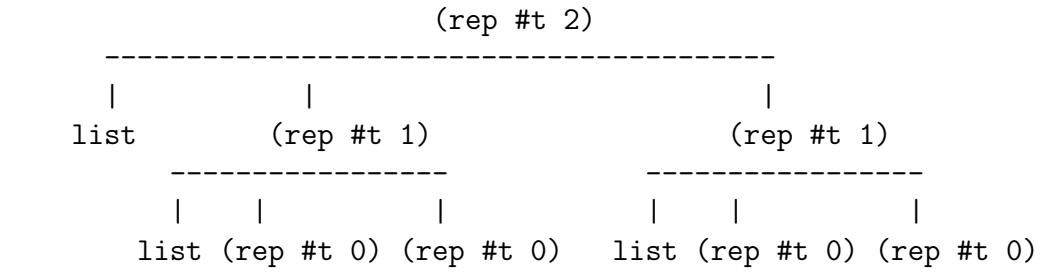
5.(c) Is the following procedure tail recursive?
Why or why not?

```
    (define (our-length lst)
      (if (null? lst)
          0
          (+ 1 (our-length (cdr lst)))))
```

No, because the value of the recursive call, (our-length (cdr lst)),
has 1 added to it before it is returned as the value of (our-length lst).

5.(d) Draw a tree of recursive calls (no return values)
for the application (rep #t 2), where the procedure rep is
defined as follows.

```
(define (rep value n)
  (if (= n 0)
      (list value)
      (list (rep value (- n 1))
            (rep value (- n 1)))))
```



5.(e) Rewrite the procedure in 5.(d) using the special form let
to avoid redundant computation.

```
(define (rep value n)
  (if (= n 0)
      (list value)
      (let ((prev (rep value (- n 1))))
        (list prev prev))))
```

6. (10 points) Evaluate the following Racket expressions.
(Remember to show work to permit partial credit!)

Example:

```
(cons 1 '(6 5)) => '(1 6 5)
```

(a)
```
(cons '(1 2) '(3 4)) => '((1 2) 3 4)
```

(b)
```
(cadr '((apple book) (cat) (doubt eel))) => '(cat)
```

(c)
```
(map (lambda (x) (cons 0 x)) '() (2) (3 5))) => '((0) (0 2) (0 3 5))
```

(d)
```
(list (+ 1 2) (list 1 2)) => '(3 (1 2))
```

(e)
```
(((lambda (x)
        (if (odd? x)
            (lambda (x) (+ x 1))
            (lambda (x) (- x 1))))  3) 12) => 13
```