Inorder Traversal

Left → Root → Right

ACB E LMN RST

sorted!

Post order on expression tree:

\[ 3 + 6 \times 2 \]

Post order: \( 3 \ 6 \ 2 \times + \)

Stack: 3 5 15 1
Red-Black Trees

Full Binary Search Trees such that

1. each node has a color: red or black
2. root is colored black
3. all leaves are colored black
4. children of red nodes are black
5. every path root → leaf has same # of black nodes

not a red-black tree (pp 5) non-leaf red-black tree!

# leaves in a non-empty full binary tree = # internal nodes + 1

k = total nodes
n = data nodes

k = 2n + 1

# internal (all b)

nodes = 2^{h+1} - 2

proof by strong induction

Suppose # black nodes in root→leaf paths is b

shortest possible path has b nodes

longest possible path has 2b - 1

2b - 1

h ≤ 2b - 1

h is O(b)

Red-black tree with black height b has ≥ 2^b - 1 nodes
$k = 2^{a} - 1$
$k + 1 = 2^{b}$

$log_2(k+1) >= b$

$b$ is $O(log k)$
and $O(log n)$
1) Do normal BST insert, color new node red (with empty black laws)

2) If new node's parent exists and is black, Done
   Else
   \[ z \in \text{node added} \]
   While \( z \) is not root and \( z \rightarrow \text{parent} \rightarrow \text{color} = \text{red} \)

\[ \begin{align*}
   &i) \quad z \rightarrow \text{parent} \rightarrow \text{color} = \text{red} \\
   &\quad \text{Do } z \rightarrow \text{parent} \rightarrow \text{color} \leftarrow \text{black} \\
   &\quad \text{Then continue}
\end{align*} \]

[other cases symmetric]

\[ \begin{align*}
   &z \rightarrow \text{root} \rightarrow \text{color} \leftarrow \text{black}
\end{align*} \]