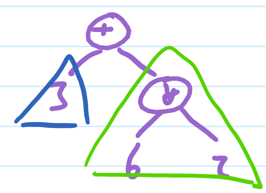
Inorder Traversal

left
root
right

ABE
AVL
CGK
LHR
MAN
MCI
⋮
Sorted !

post order on expression tree

3 + 6 * 2



post order 3 6 2 * +

stack ~~2~~ ~~6~~ 3 ~~12~~ ~~3~~ 15
\+ *

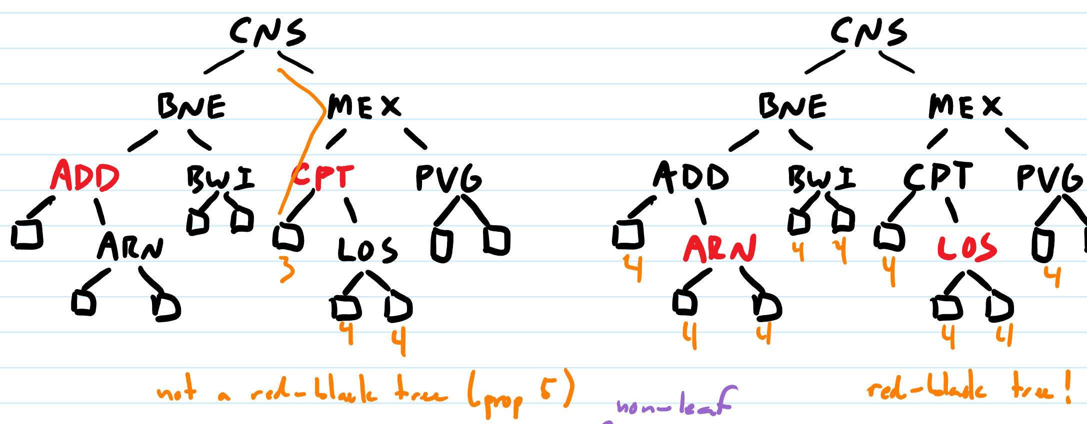
Red-Black Trees

Full Binary Search Trees such that

all nodes 0 or 2 children

1) each node has a color: red or black
2) root is colored black
3) all leaves are colored black
4) children of red nodes are black
5) every path root → leaf has same # of black nodes

black height



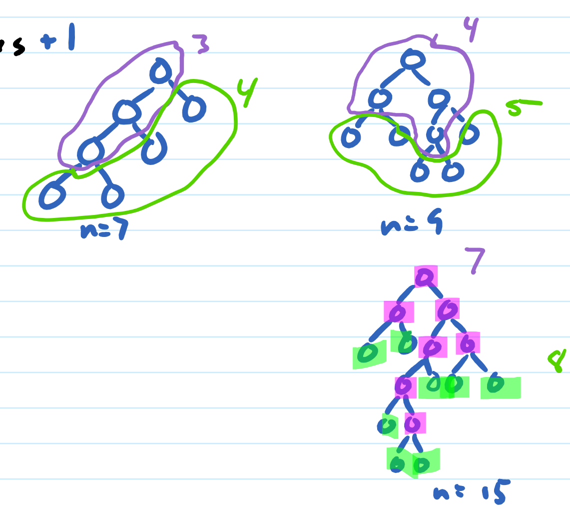
leaves in a non-empty full binary tree = # internal (non-leaf) nodes + 1
(non-leaves)

k = total nodes
n = data nodes

$k = 2n+1$

if time was $O(\log 2n+1)$

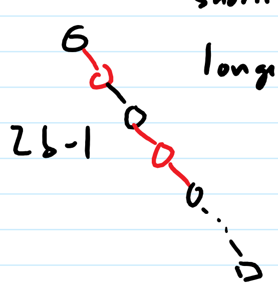
$= O(\log n)$



Suppose # black nodes in root → leaf paths is b

shortest possible path has b nodes

longest possible path has $2b-1$

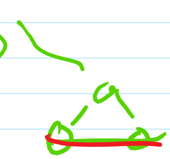


$h \le 2b-1$

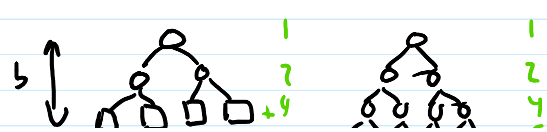
h is $O(b)$

internal (data) nodes = $\frac{\#\text{ nodes}-1}{2}$

prove by structural induction

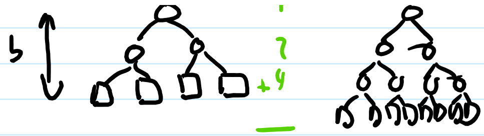


Red-black tree with black height b has $\ge 2^b-1$ nodes



$k \ge 2^b - 1$

$k+1 \ge 2^b$



$$1 + 2 + 4 = 7$$

$$1 + 2 + 4 + 8 = 15$$

$k \geq 2^b - 1$

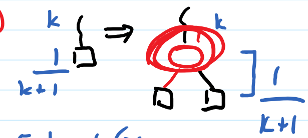
$k + 1 \geq 2^b$

$\log_2 k+1 \geq b$

b is $O(\log k)$

and $O(\log n)$

1) Do normal BST insert, color new node red (with empty black leaves)



prop 5 true before → true after

2) If new node's parent exists and is black, DONE

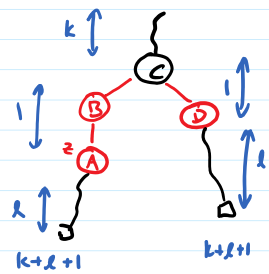
Else

$z \leftarrow$ node added

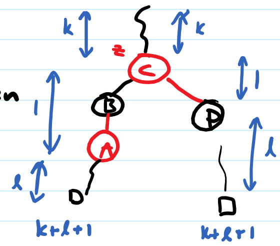
red node we're worried about

while z is not root and $z \to parent \to color$ == red

if

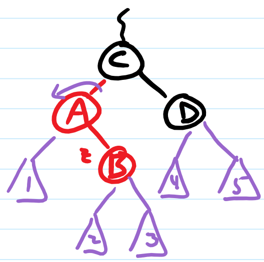


then

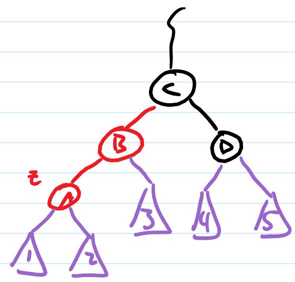


else

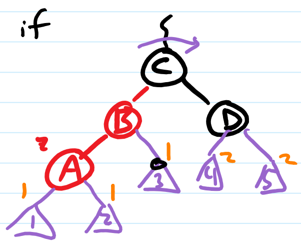
if



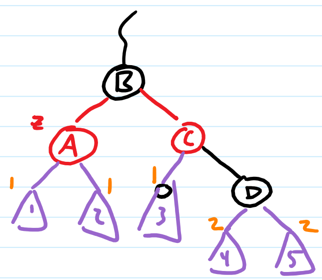
then



if



then



STOP!
(no more red/red)

[other cases symmetric]

$t \to root \to color \leftarrow$ black