

# Breadth-First Search

BFS (G, s)

```

for each vertex u in graph_vertices(G)
  color[u] ← WHITE
  d[u] ← infinity
  pred[u] ← NIL
  
```

```

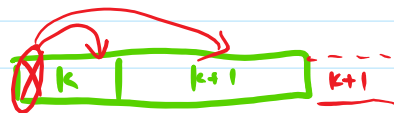
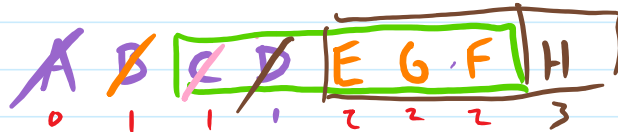
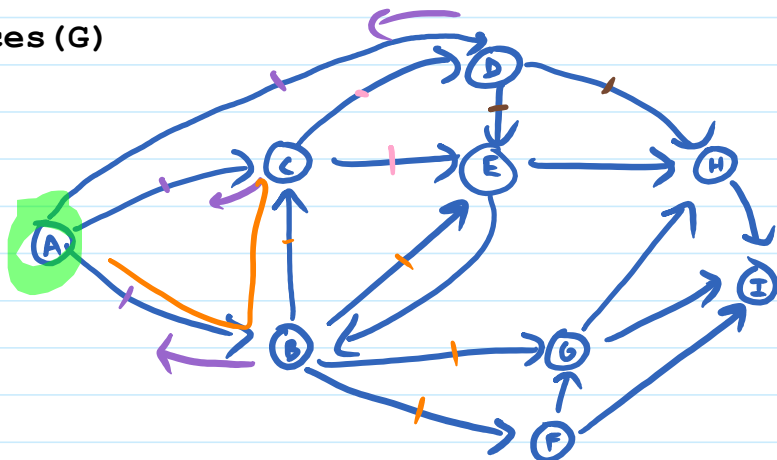
color[s] ← GRAY
d[s] ← 0
pred[s] ← NIL
  
```

```

Q ← [s]
  
```

```

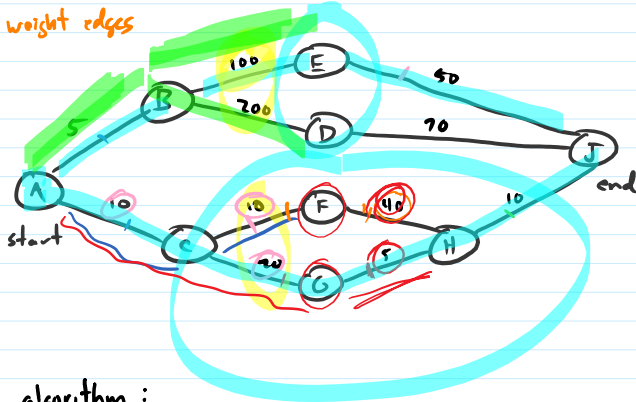
while not queue_is_empty(Q)
  u ← queue_dequeue(Q)
  for each v in graph_adjacent(u)
    if color[v] = WHITE
      color[v] ← GRAY
      d[v] ← d[u] + 1
      pred[v] ← u
      queue_enqueue(Q, v)
  color[u] ← BLACK
  
```



invariant: verts on queue are in order of ↑ distance with those @ end at most one edge further than those at start

Shortest Paths in Weighted Graphs

assume no neg weight edges



A	0
B	5
C	10
E	105 95
D	205 115
F	10
H	20 30
I	45

A  
A  
J  
J  
C  
C  
G  
H

Dijkstra's algorithm :

Dijkstra(G, s)

```

Q <- pqueue_create()
for each vertex u in graph_vertices(G)
  d[u] <- infinity
  pred[u] <- NIL
  pqueue_add(Q, u, d[u])
  
```

```

d[s] <- 0
pred[s] <- NIL
pqueue_update(Q, s, 0)
  
```

adj list

```

while not pqueue_is_empty(Q)
  u <- pqueue_extract_min(Q)
  for each v in graph_adjacent(u)
    if pqueue_contains(Q, v) and d[u] + graph_weight(G, u, v) < d[v]
      d[v] <- d[u] + graph_weight(G, u, v)
      pred[v] <- u
      pqueue_update(Q, d[v])
  
```

$O(n+m)$

$O(1)$  each  
 $O(m)$  time }  $O(m)$  total

$O(n^2)$  overall

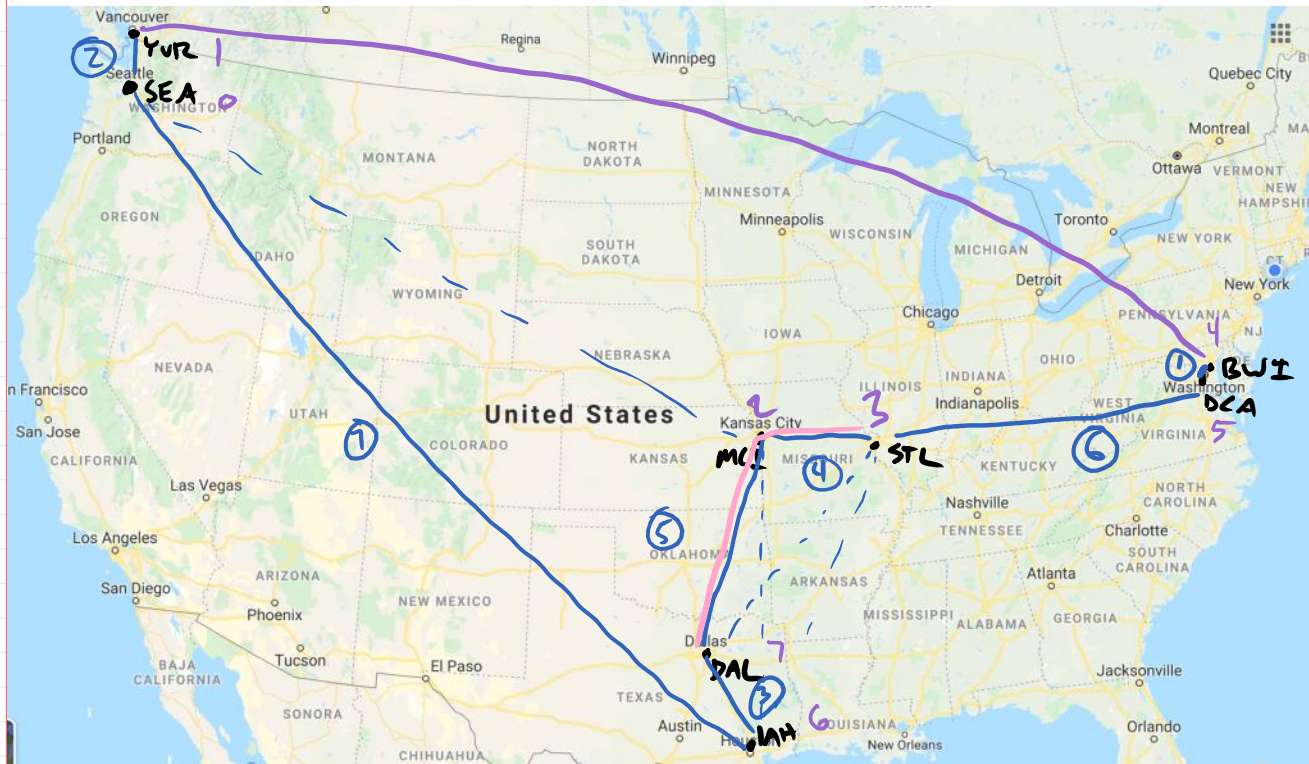
$O(n)$  each  
 $O(n)$  times }  $O(n^2)$  total

Priority Queue : items + priorities  
 add (item, priority)  
 extract\_min() : removes item w/ lowest priority  
 update (item, priority): changes priority of item

unsorted array  
 $O(1)$   
 $O(n)$   
 $O(1)$

heap  
 $O(\log n)$   
 $O(\log n)$   
 $O(\log n)$

# Greedy TSP



for each pair of cities in order of increasing distance  
if not connected yet and degree both  $\leq 1$   
add edge to tour

BFS

find vertices  $u, v$  with degree 1

find path  $u \rightsquigarrow v$   
add edge  $(v, u)$