

Expected Time vs Amortized Time

$$1 + \frac{1}{6}$$

```
int roll5()
{
  int roll;
  do
  {
    roll = roll6();
  } while (roll == 5);
  return roll;
}
```

X = number of calls to roll 6 in a call to roll 5

$$E[X] = \frac{5}{6} \cdot 1 + \frac{1}{6} \cdot \frac{5}{6} \cdot 2 + \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 3 + \left(\frac{1}{6}\right)^3 \cdot \frac{5}{6} \cdot 4 + \dots = \frac{6}{5} \text{ } O(1) \text{ expected}$$

$$P(n \text{ calls to roll 5 take } > 2n \text{ calls to roll 6}) > 0$$

$$P(n \text{ calls to roll 5 take } > 10n \text{ calls to roll 6}) > 0$$

$$P(n \text{ calls to roll 5 take } > n^2 \text{ calls to roll 6}) > 0$$

amortized $O(1)$

$$P(n \text{ consecutive adds require } > 4n \text{ copies}) = 0$$

$$\frac{4n}{n} = 4$$

add worst case $O(n)$ ←
 amortized $O(1)$ ←
 ↑
 average

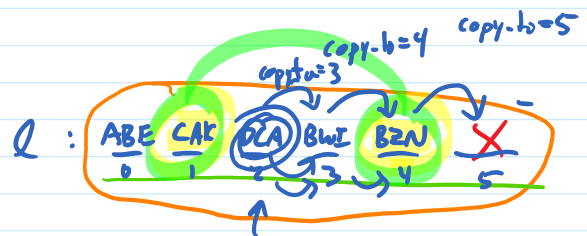
List Implementations

	standard array
<u>size</u>	$O(1)$
<u>get</u>	<u>$O(1)$</u>
add beginning arbitrary end	$O(n)$ $O(n)$ $O(1)$ amortized
remove beginning arbitrary end	$O(n)$ $O(n)$ $O(1)$ amortized

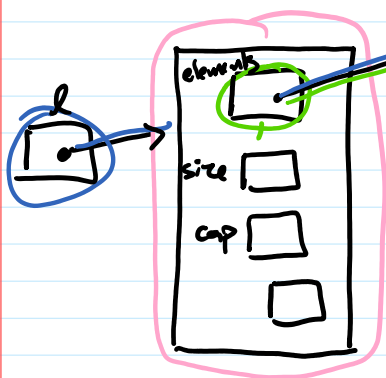
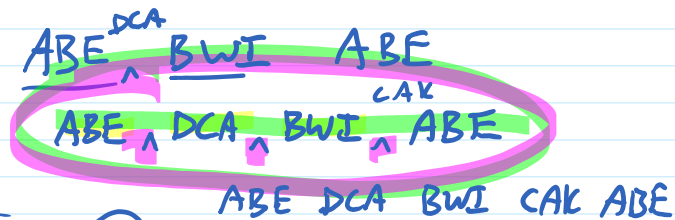
contains / find $O(n)$

↓ does list contain element?
↓ gives index of 1st occurrence

add(l, LHR, $\frac{0}{2}$)



remove(l, 3)



$(l).elements[copy-to] = l \rightarrow elements[copy-to-1]$



i [2] to-add [99]

resize after 5th add
 | remove ... resize
 | add ... resize
 | remove ... resize

Wraparound array

