

20144987 4 4 4 CLT OAK X 17 1.00 X
 20144756 2 4 4 CLT BOS X US 1.00 X
 201441020 2 4 4 CLT MCO X US 1.00 X
 201442094 3 4 4 CHS CLT - 16 1.00 X
 20144214 3 4 4 IAH CLT - US 1.00 X
 201441020 1 4 4 AVL CLT - 16 1.00 X
 201441110 4 4 4 PHX OAK X US 1.00 X
 20144794 3 4 4 CLE CLT - 16 1.00 X
 201441578 4 5 4 BNA PHL - YX 1.00 X
 201442030 4 4 4 DCA BHM X US 1.00 X
 201441020 3 4 4 MCO CLT - US 1.00 X

CLT 65207 % 8 = 0
 OAK 78009 % 8 = 1
 BOS 65958 % 8 = 6
 MCO 76153 % 8 = 1

key → int
 hash fxn

AVL 65207 % 8 = 0
 CHS 66702 % 8 = 6
 IAH 72240 % 8 = 0
 PHX 79200 % 8 = 0
 → % 16 = 0

hash table

key	value
0	
1	OAK 1
2	
3	CLT 2
4	
5	
6	BOS 1
7	X

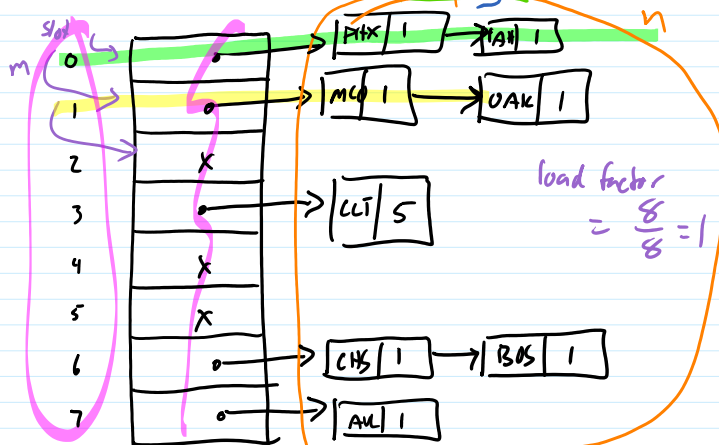
contains-key: compute hash(key) % size
 if NULL at that location, return NO
 if something there, do comparison

collision w/ BOS
 ↓
 resolve collisions on put

colliding keys saved in same linked list

hash table w/ chaining

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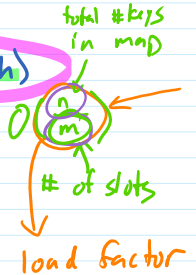


CLT
 AVL
 IAH
 PHX
 OAK
 CLE
 BOS
 MCO

contains key: compute hash(key) % # slots
 search linked list at that index (sequential search)

get: compute hash(key) % # slots
 search linked list at that index (sequential search)
 return value in node

put: compute hash(key) % # slots
 search linked list at that index (sequential search)



hash fxn should distribute keys evenly

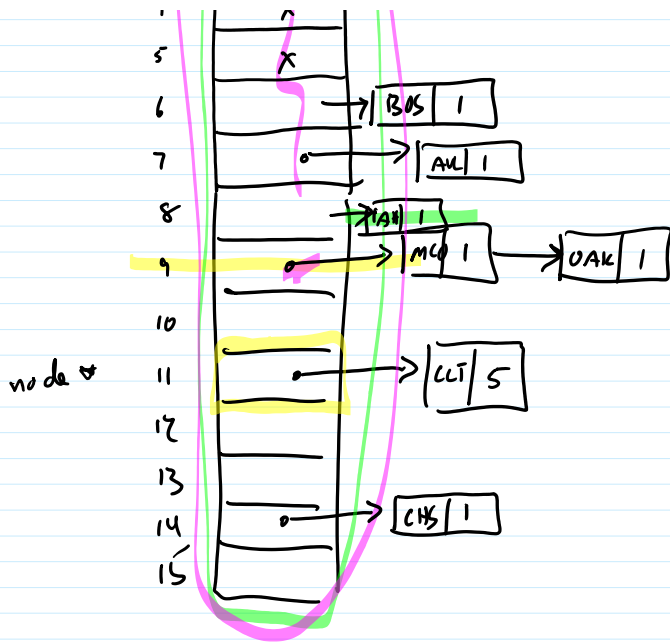


contains-key (OAK)

hash(OAK) = 78009

$78009 \% 16 = 9$

(expected) running time = $O(\text{load factor})$



expected running time = $O(\text{load factor})$
 if $m \geq n$
 = $O(1)$

worst case $O(n)$
 (all keys collide in same slot)

```

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201441578 4 5 4 BNA PHL - YX 1.00 X
201442030 4 4 4 DCA BHM X US 1.00 X
201441020 3 4 4 MCO CLT - US 1.00 X
  
```

hash fun
 CLT 3
 AVL 7
 IAH 21
 PHX 805
 → OAK 457
 CLE
 BOS
 MCO

21% $q=5$
 805% $q=5$ collision

hash table w/ open addressing
 linear probing

key	value
0	OAK 1
1	
2	
3	CLT 2
4	
5	IAH 1
6	PHX 1
7	AVL 1

contains key put get
 $O(n)$

- 1) compute hash
- 2) compute %
- 3) seq. search through slots until key found or empty or wrapped all the way around

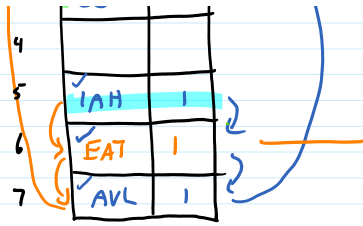
if $\alpha \leq \frac{1}{2} \rightarrow O(1)$ expected time

key	value
0	✓ OAK 1
1	
2	
3	✓ CLT 2
4	
5	✓ IAH 1

hash fun
 CLT 3
 AVL 7
 IAH 21
 PHX 805
 → OAK 457
 EAT 8885

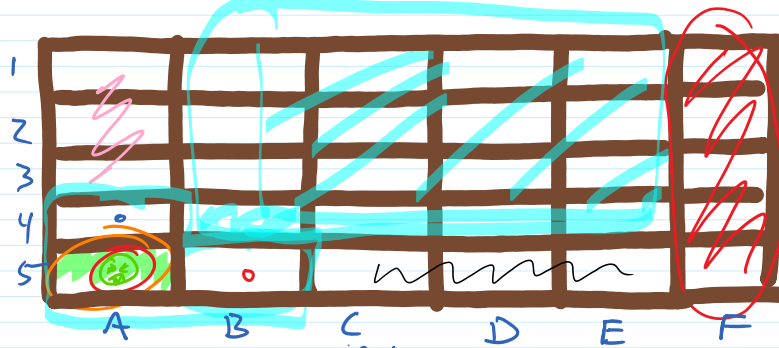
21% $q=5$
 805% $q=5$

EAT 8885



Play on $m \times n$ grid. Take turns selecting remaining cell, remove all above and to right

Last move loses.
(misere)

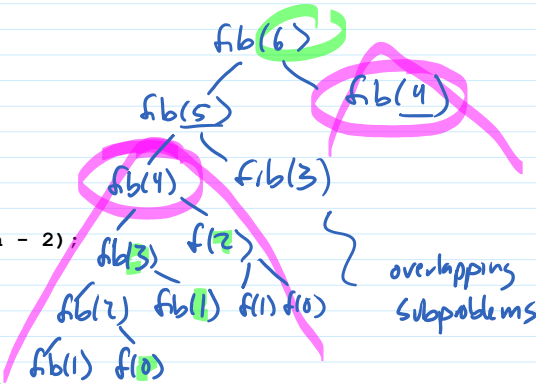


outcome-class(p)

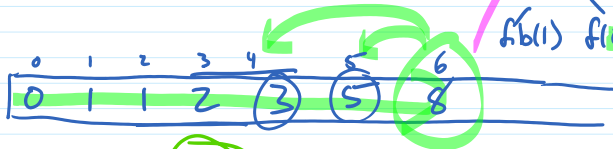
if p is end of game
return value according to rules win
else
S ← positions reachable in 1 move from p
if S contains a losing position
return win, and corresponding max
else
return LOSE

```
long fib_rec(int n)
{
    if (n < 2)
    {
        return n;
    }
    else
    {
        return fib_rec(n - 1) + fib_rec(n - 2);
    }
}
```

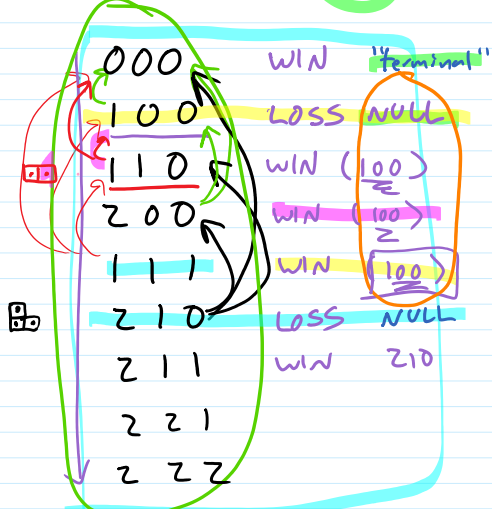
exponential running time!



overlapping subproblems

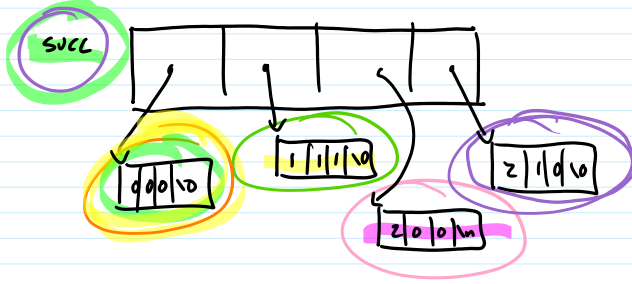
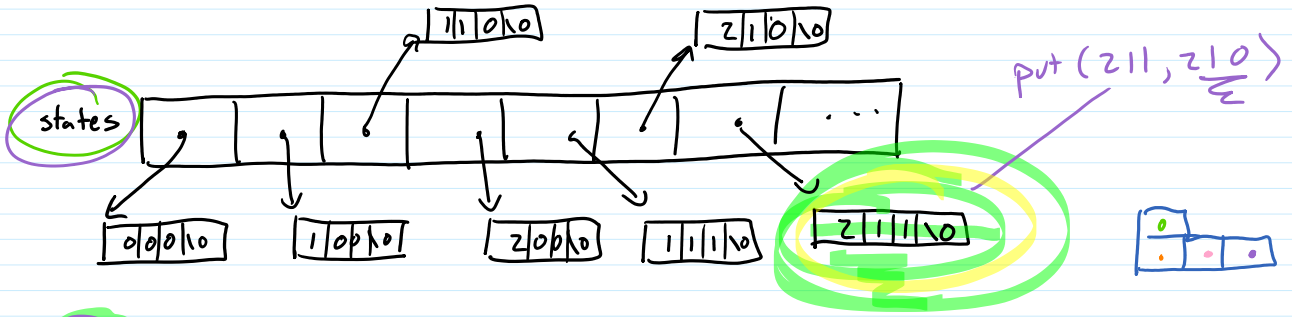


dynamic programming (replace recursion w/ array)



map keys: string representation of state
values: NULL (losing position)
string rep of state to map to for winning max

z z z



000
 100
 110
 200
 111
 210
 211
 221
 222