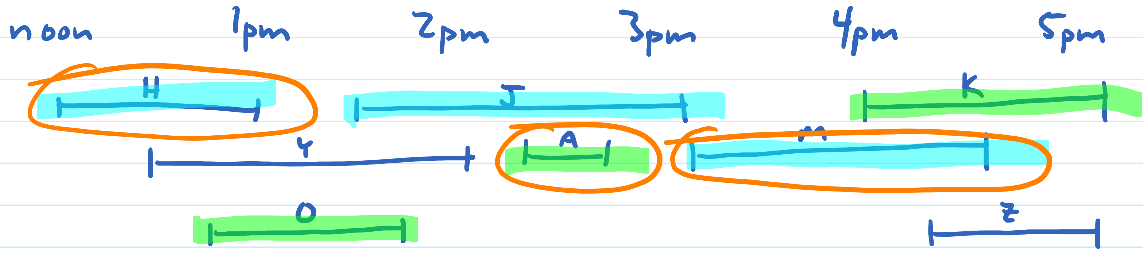

noon
1pm
2pm
3pm
4pm
5pm
H
J
K
Y
A
M
O
Z

Invariants

for $i = 0$ to $n-1$
$A[i] = i$

```
SelectionSort(A)
   for i = 0 to n-2
      find min among A[i],...,A[n-1]
      swap min with A[i]
```

$i = 0$
while $i \le n-2$
$i{+}{+}$

3 1 2 5 4

1 | 3 2 5 4

1 2 | 3 5 4

1 2 3 | . . . . .

INVARIANT: $A[0] \le A[1] \le \cdots \le A[i-1]$

is $A[0] \le A[1] \le \cdots \le A[i-1] \le$ min among $A[i], \ldots, A[n-1]$

?? = min of $n-i$ largest = $(i+1)^{st}$ smallest = $A[i]$

and $A[0], \ldots, A[i-1]$ are i smallest from $A_0[0], \ldots, A_0[n-1]$
and $A[i], \ldots, A[n-1]$ are $n-i$ largest from

```
InsertionSort(A)
   for i = 1 to n-1
      insert A[i] into correct location among A[0],...,A[i-1]
```

INVARIANT: $A[0] \le A[1] \le \cdots \le A[i-1]$

3 | 1 2 5 4

1 3 | 2 5 4

1 2 3 | 5 4

Interval Scheduling

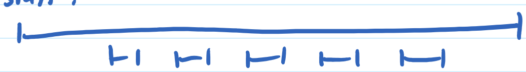
Given intervals labelled $1, \dots, n$ w/ interval i's start, finish $= s(i), f(i)$,
find largest set S of pairwise compatible intervals

no pair of selected intervals overlaps,
so $i, j \in S$ $i \neq j$ $s(i) \le f(i) \le s(j) \le f(j)$
or $s(j) \le f(j) \le s(i) \le s(j)$

Greedy: choose shortest?

shortest gives 2 optimal has 3 :(

choose 1^{st} to start?

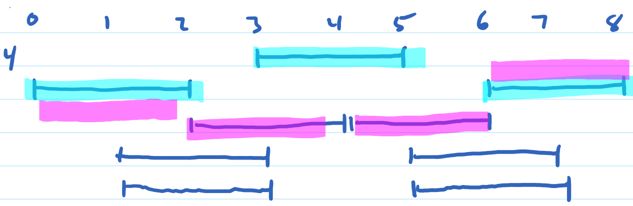


:(

last to end?

fewest overlaps?

choose 3; opt gets 4



from Jyh-huw Yeh, Boise State

Choose 1^{st} to end

$R \leftarrow \{1, \dots, n\}$ sorted by finishing time next $\leftarrow 1$ $O(n \log n)$
$A \leftarrow \emptyset$ $r \leftarrow 0$
while $R \neq \emptyset$
  choose $i \in R$ to minimize $f(i)$
  $A \leftarrow A \cup \{i\}$ $r \leftarrow r+1$
  remove from R those incompatible with i
return A

while next $\le n$
  $A = A \cup \{\text{next}\}$
  next $\leftarrow$ next $+1$
  while $s(\text{next}) < f(r)$
    next $\leftarrow$ next $+1$

INVARIANT: condition that is true before every iteration of loop (before test)

Let O be opt solution $j_1, j_2, \dots, j_k$ in order of $\uparrow f$
for $i = 1, k-1$, $s(i) \le f(i) \le s(i+1) \le f(i+1)$

INV: after r iterations, $f(a_r) \le f(j_r)$
and $s(\text{next}) \ge f(a_r)$ and for $p < \text{next}$, $s(p) < f(a_r)$
interval next starts after last one we added finishes
can't add any intervals before next

Suppose INV T after $r-1$ iterations (will show T after r iterations)

Suppose after $r-1$ iterations $f(a_{r-1}) \le f(j_{r-1})$
and $s(\text{next}) \ge f(a_{r-1})$ and $\forall p < \text{next}$, $s(p) < f(a_{r-1})$
[want $f(\text{next}) \le f(j_r)$]

suppose $j_r < \text{next}$. then $s(j_r) < f(a_{r-1}) \le f(j_{r-1})$

iterations
(will show T after
r iterations)
[want $f(a_r) \le f(j_r)$]

suppose $j_r < next$. then $s(j_r) < f(a_{r-1}) \le f(j_{r-1})$

contradicts $\mathcal{O}$ being ordered by f

$\therefore$ $next \le j_r$ (reject assumption that led to $\Rightarrow\Leftarrow$)

so $f(next) \le f(j_r)$ (ordered by ↑ f)

and $f(a_r) \le f(j_r)$, preserving the 1st part of the invariant
(code chooses $a_r = next$)

(inner while loop maintains the rest of the invariant)

Our loop terminates after k iterations (so outputs a set of same size as $\mathcal{O}$)

- after $k-1$ iterations, we have $f(a_{k-1}) \le f(j_{k-1})$ (invariant)

and $f(j_{k-1}) \le s(j_k) \le f(j_k)$ ($\mathcal{O}$ is sorted, compatible)

so $f(a_{k-1}) \le s(j_k)$

$\therefore$ $next \le j_k$ (invariant - if $j_k < next$ then $s(j_k) < f(a_{k-1})$)

$j_k \le n$ (only n intervals to choose)

so $next \le n$ and loop iterates for k^{th} time

(can't iterate $> k$ times b/c then A is better than optimal)
(and intervals in A are pairwise compatible - show by adding that to the invariant)