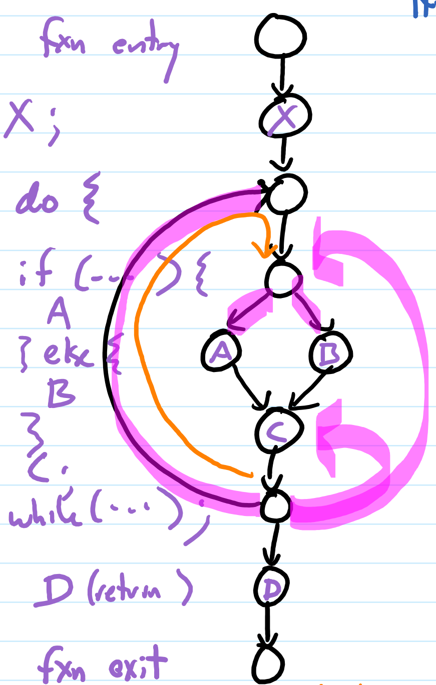
Program Dependence Graphs



acyclic control dependence graph in this case

Hyp: Dependence Hampers Testability

Control Flow Graph: vertex = line of code
edge (u,v) if v can immediately follows u in execution

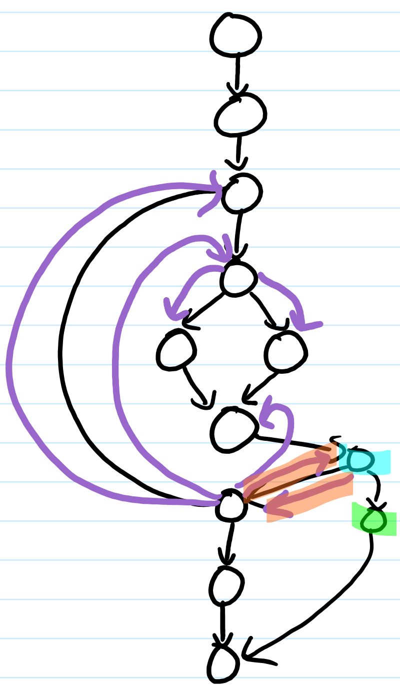
Post-Domination: a post-dominates b if every path b ⇝ exit goes through a
(strict if $a \neq b$)

D post-dominates all except exit

C post-dominates do

A, B don't post dominate do

Control Dependence: a is dependent on b if
1) a does not strictly post-dominate b
2) ∃ path b ⇝ a s.t. all verts on path after b are post-dom by a



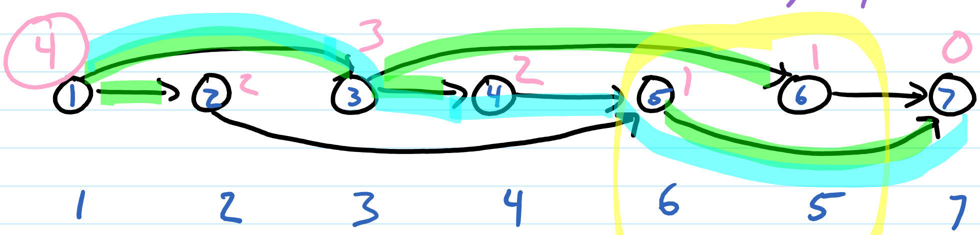
if (....)
return

more dependence edges
↓
cycles!

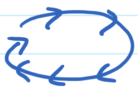
Given directed graph G, find simple path with most edges in G

In general thought to be hard (decision problem is NP-complete)

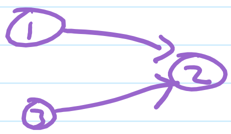
Directed Acyclic Graph: $O(n+m)$ 1) topological sort (all edges →)
2) dynamic programming



← reverse order of finish after DFS



LONG(n) = length of longest path that starts at vertex n
numbered in order of topo sort



$$\begin{cases} = 0 & \text{if no outgoing edges} \\ = \max\limits_{(v_n, v_m)} \text{LONG}(m) + 1 \end{cases}$$

1 3→2

3→1 2

Compute LONG(n) in reverse order of topo sort

Longest Common Subsequence

X: A C C G T A | A C | T

Y: G T C T C T A G | A

CCTAA is LCS of $X_{1\ldots8}, Y_{1\ldots9}$
and of $X_{1\ldots7}, Y_{1\ldots9}$

CCTA is LCS of $X_{1\ldots6}, Y_{1\ldots8}$

$\text{lcs-len}(i,j)$ = Length of LCS of $X_{1\ldots i}$ and $Y_{1\ldots j}$

$O(n \cdot m)$ entries
$O(1)$ to compute each
$O(n \cdot m)$ overall

$$= \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ 1+\text{lcs-len}(i-1,j-1) & \text{if } x_i = y_j \\ \max(\text{lcs-len}(i-1,j), \text{lcs-len}(i,j-1)) & \text{otherwise} \end{cases}$$

THM: Let Z be LCS of X and Y

Then if $x_m = y_n$ then $Z_{1\ldots k-1}$ is an LCS of $X_{1\ldots m-1}, Y_{1\ldots n-1}$

and if $x_m \neq y_n$ and $z_k \neq x_m$ then Z is an LCS of X_{m-1}, Y

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Proof: 1) Suppose Z is LCS of X, Y and $x_m = y_n$

Suppose further that $Z_{1\ldots k-1}$ is not LCS of $X_{1\ldots m-1}, Y_{1\ldots n-1}$;

Then $Z' \cdot x_m$ is

$\text{len}(Z' \cdot x_m) =$

2) Suppose Z is LCS of X, Y, $z_k \neq x_m$ and $x_m \neq y_n$

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Z is CS of $X_{1\ldots m-1}, Y$:

Z is <u>L</u>CS of $X_{1\ldots m-1}, Y$:

3) similar

$$\text{lcs-len}(i,j) = \begin{cases} \end{cases}$$

		A	C	C	G	T	A	A	C	T
	0	0	0	0	0	0	0	0	0	0
G	0	0	0	0	1	1	1	1	1	1
T	0	0	0	0	1	2	2	2	2	2
C	0	0	1	1	1	2	2	2	3	3
T	0	0	1	1	1	2	2	2	3	4
C	0	0	1	2	2	2	2	2	3	4
T	0	0	1	2	2	3	3	3	3	4
A	0	1	1	2	2	3	4	4	4	4
G	0	1	1	2	3	3	4	4	4	4
A	0	1	1	2	3	3	4	5	5	5