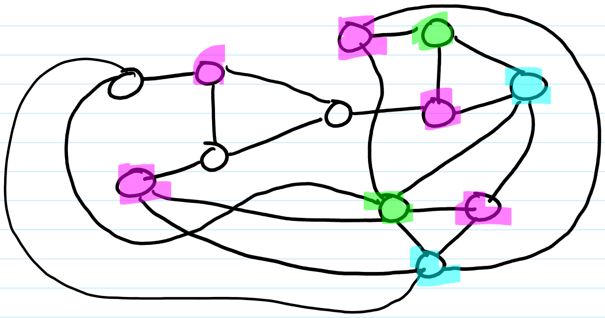
3-COLORING

Coloring of an undirected graph G: assignment of $\{1, \dots, k\}$ to verts s.t. no edge has same value at both ends

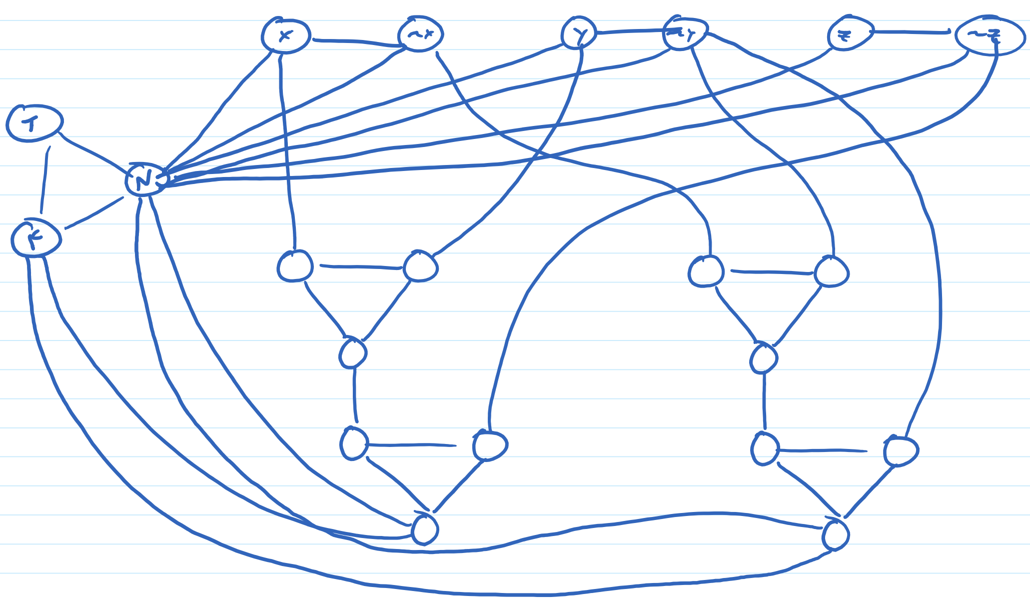


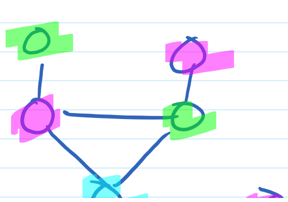
Prove: 3-COLORING is NP-complete

1) 3-COLORING $\in$ NP

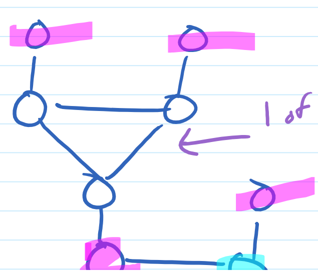
2) 3-SAT $\leq_p$ 3-COLORING [Goal: given 3-CNF formula φ, create G s.t. φ is satisfiable $\Leftrightarrow$ G has 3-coloring]

$$(x \vee y \vee \sim z) \wedge (\sim x \vee \sim y \vee \sim y)$$



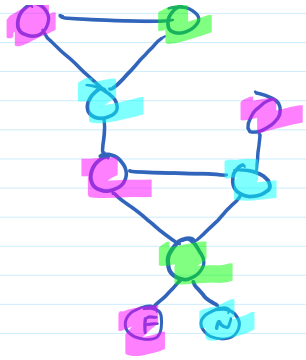


if widget is connected to ≥ 1 T then it is 3-colorable (check the other cases)

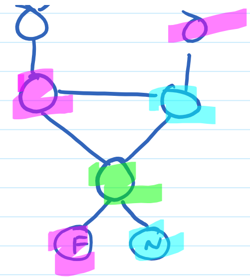


if widget connected to 3F, not 3-colorable

1 of 3 must be F

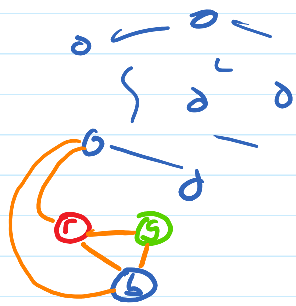


if widget is attached
to ≥ 1 T then it
is 3-colorable
(check the other cases)



Non-Decision Problems

If 3-COLORING ∈ P then can find a 3-coloring in poly-time



FIND-3-COLORING(G)

Add clique r, g, b to G to get G'

for each vertex v in G

Inv: G' is 3-colorable ⇕ G is 3-colorable

$G_r \leftarrow G'$ w/ (v,g) (v,b)
$G_b \leftarrow G'$ w/ (v,r) (v,g)
$G_g \leftarrow G'$ w (v,r) (v,b)

polynomial calls to 3-COLOR, polynomial other work

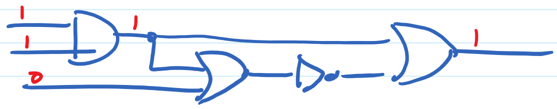
```
if 3-COLOR(G_r)
    set color[v] = r
    G' ← G_r
else if 3-COLOR(G_b)
    set color[v] = b
    G' ← G_b
else if 3-COLOR(G_g)
    set color[v] = g
    G' ← G_g
else
    return NIL

return color
```

CIRCUIT-SAT is NP-complete (Cook-Levin Thm)

really SAT is NP-c but similar idea

CIRCUIT-SAT: given combinational circuit, is there an assignment of 0/1 to inputs to make output 1

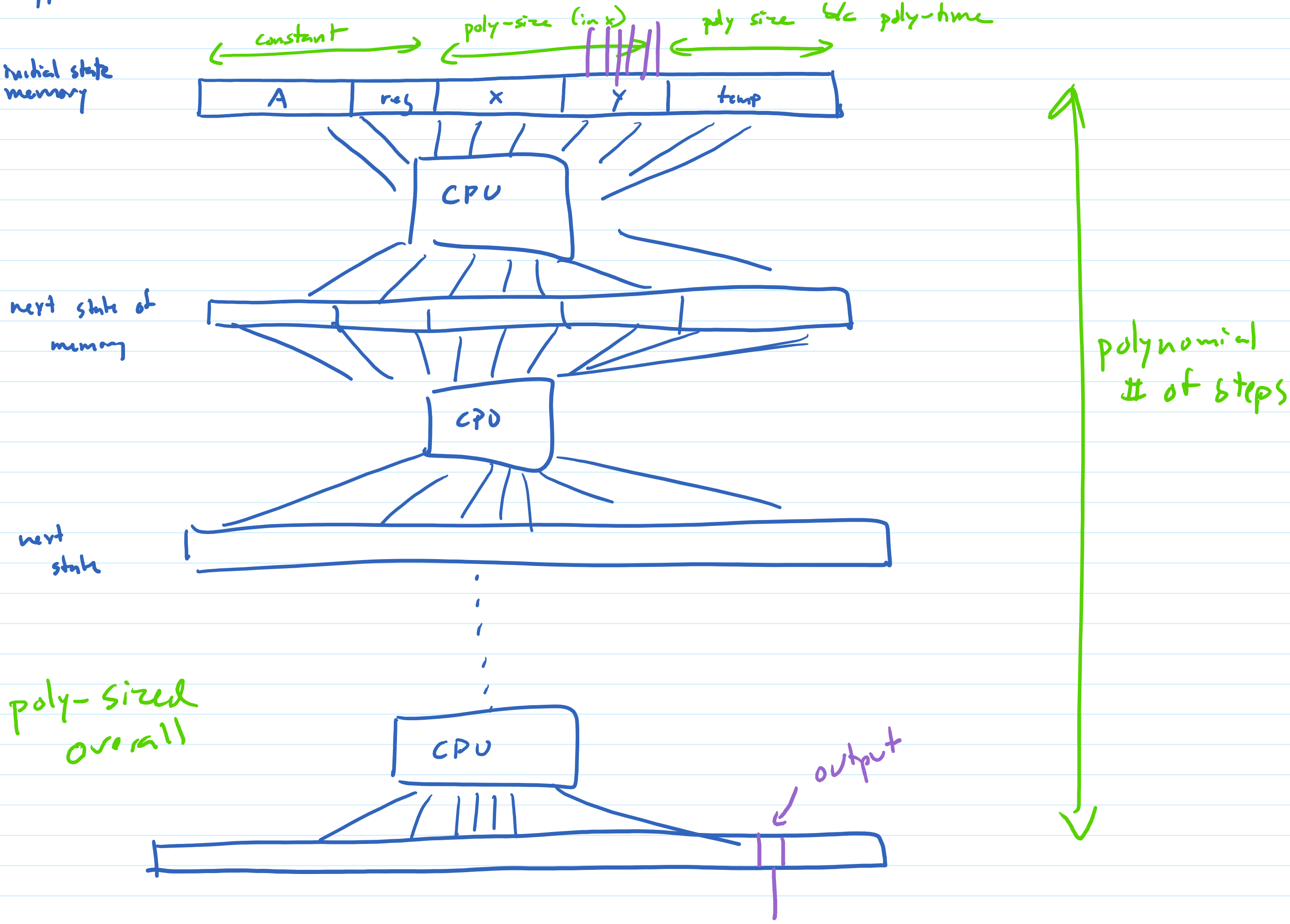
combinational: and/or/not gates, no output feeds back into input (no memory)



1) CIRCUIT-SAT $\in$ NP   evidence is satisfying assignment

2) $\forall L \in NP$, $L \leq_p$ CIRCUIT-SAT

Suppose $L \in NP$. So L has poly-time verification alg A



$x \in L \rightarrow \exists y$ s.t. $A(x,y) = YES \rightarrow$ that y is satisfying input to circuit

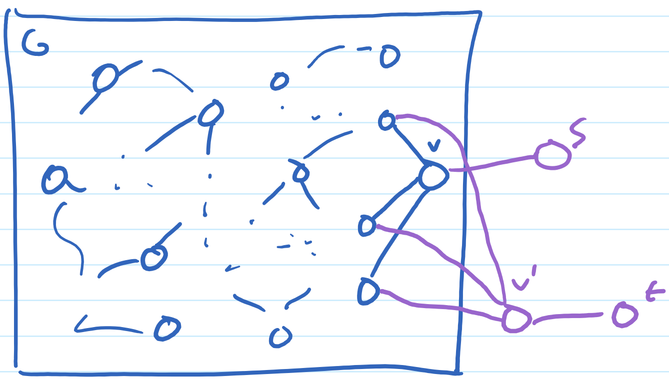
$x \notin L \rightarrow \forall y, A(x,y) = NO \longrightarrow$ no satisfying input to circuit

UNDIR-HC and UNDIR-HP

UNDIR-HP is NP-complete

1) UNDIR-HP $\in$ NP evidence is the path
2) UNDIR-HC $\leq_p$ UNDIR-HP

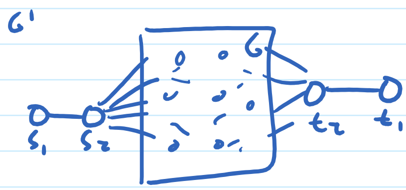
Goal: Given G, construct G' s.t. G has a HC $\longleftrightarrow$ G' has a HP



so if HP in G', it must be (wlog) $s, \underbrace{v, u_1, \ldots, u_{n-1}, v'}_{\text{HP in } G \text{ (replacing } v' \text{ w/ } v)}, t$

if HC $v, u_1, \ldots, u_{n-1}, v$ in G then $s, v, u_1, \ldots, u_{n-1}, v', t$ is HP in G'

$\overbrace{s_1, s_2, t_1, t_2 - \text{HP}}^{\text{given } G \text{ and } 1^{st}\ 2,\ \text{last } 2 \text{ verts, is there an HP?}}$ is NP-complete



G has HP $\longleftrightarrow$ G' has HP that starts s_1, s_2 and ends t_2, t_1