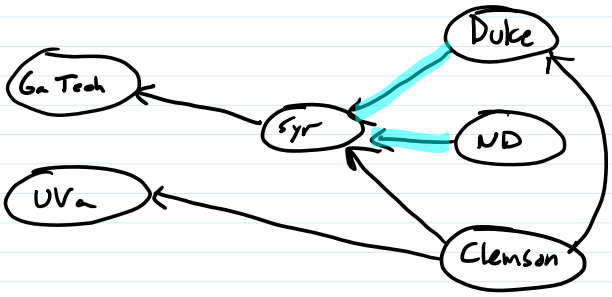
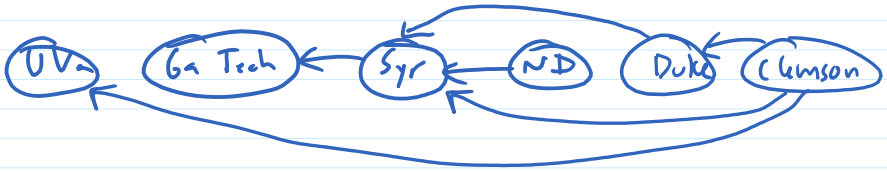
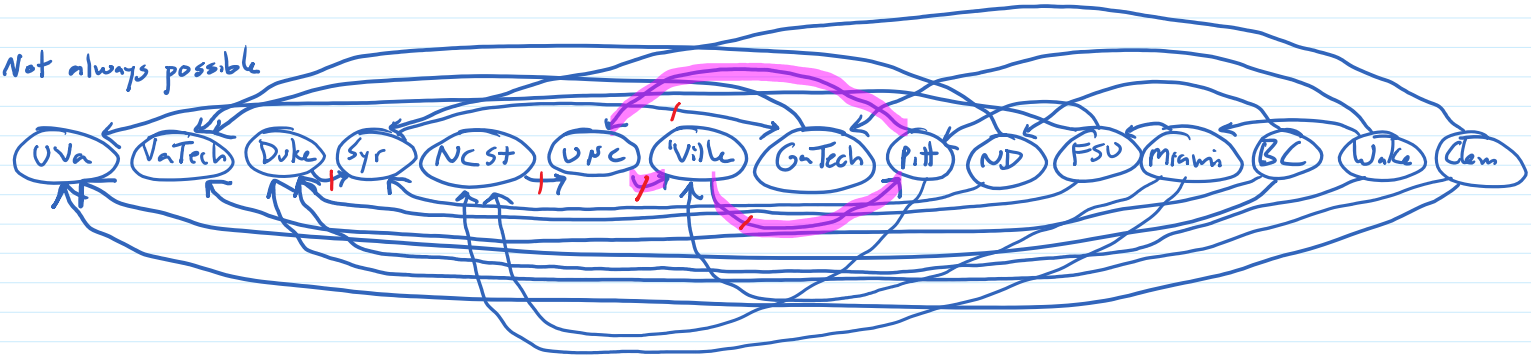
Feedback Arc Set



Order vertices so all edges go ←



Not always possible



find ordering of vertices to minimize "wrong way" edges
(5 total in this example)

Brute Force: try every ordering
for each, compute number of upsets
keep track of running minimum

n! orderings

353! ≈ $5 \cdot 10^{747}$

Feedback Arc Set is NP-complete

↓

the hardest of a certain set of problems;
for these hardest problems we have

1) no known efficient soln
2) no proof that no efficient soln exists

Mars Rovers — $2.5B budget

6 years (so far)

~$7.5M / week

software updates

2 GB flash

~10 Kbps https://marsmobile.jpl.nasa.gov/msl/mission/communicationwithearth/data/

~18 days

NASA is a big sponsor of formal verification

https://shemesh.larc.nasa.gov/fm/

Therac-25

radiation therapy machine

hardware interlocking mechanism replaced with faulty software

3 deaths

formal verification important for safety- and mission-critical systems

Asymptotic Running Time

Selection sort is $O(n^3)$

$T(n)$ is $O(f(n))$ means there are constants $n_0 \geq 0$, $c > 0$ s.t. for all $n \geq n_0$, $T(n) \leq c \cdot f(n)$

selection sort does $\frac{n(n-1)}{2}$ comparisons

informal - for large enough n $T(n) \leq f(n)$ (ignoring multiplicative constants)

$\forall n \geq 10$, $\frac{n(n-1)}{2} \leq 1 \cdot n^3$

$\Omega(f(n))$: there are constants $n_0 \geq 0$, $c > 0$ s.t. for all $n \geq n_0$, $T(n) \geq c \cdot f(n)$

$\Theta(f(n))$: there are constants $n_0 \geq 0$, $c_1, c_2 > 0$ s.t. for all $n \geq n_0$, $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$

so the running time of selection sort is $O(n^3)$ (and $O(n^7)$ and $O(n^4)$, $O(n^5)$, ...)

more specific to say "the running time of selection sort is $\Theta(n^2)$"

similarly "the worst case running time of quicksort is $\Theta(n^2)$"

is better (more specific) than "quicksort is $O(n^7)$"

still true, just less specific than we could be