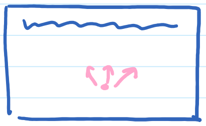
Seam Carving

https://www.youtube.com/watch?v=6NcIJXTlugc

$M[i,j]$ = cost of min-cost seam from top to row i, col j

want $M[i,h]$

$$= \begin{cases} E(i,j) & \text{if } i=1 \\ \min\limits_{d=-1,0,1} M[i-1,\, j+d] + E[i,j] & \text{otherwise} \end{cases}$$



for $j=1$ to w
  $M[1,j] = E[1,j]$
for $i=2$ to h
  for $j=1$ to w
    $M(i,j) =$

$w \cdot h$ entries
$\Theta(1)$ per entry, so $\Theta(w \cdot h)$ tot

E = Energy

1	1	1	1	4	1	2
2	3	6	5	1	0	3
3	2	4	3	2	2	0
4	0	2	1	6	5	3
5	1	1	0	2	1	0

9 4 4

1	1	1	1	4	1	2
2	4	7	6	2	1	4
3	6	8	5	3	3	1
4	6	7	4	9	6	4
5	7	5	4	6	5	4

← want min value in this row

Vendor Scheduling

	Week 1	2	3	4	5	
Brooklyn	6	3	10	4	1	$\boxed{M=3}$ moving cost
Stratford	4	6	3	8	20	$M=10$
	S	B	B >	B	B B	
	B	B	B	S	S	44

https://play.golang.org/p/YvmAsxoYER

not opt for 2 week
but B opt for 2 week
given end in B

B_i = profit in Brookly in week i
S_i = profit in Stratford in week i

$OPT_B(i)$ = max profit for weeks 1...i ending at B

$OPT_S(i)$ = " " S

$$OPT_B(i) = \begin{cases} B_1 & \text{if } i=1 \\ \max\left(\underbrace{OPT_B(i-1)+B_i}_{\text{stay in B}},\ OPT_S(i-1)+B_i-M\right) \end{cases}$$

$$OPT_S(i) = \begin{cases} S_1 & \text{if } i=1 \\ \max\left(OPT_S(i-1)+S_i,\ OPT_B(i-1)+S_i-M\right) \end{cases}$$

$\Theta(1)$ to compute each entry

$\Theta(n)$ entries (n = # weeks)

$\Theta(n)$ total

OPT_B	B_1							
OPT_S	S_1							

A C C G T A A C T
G T C T C T A A A

X : A C C G T A A C T

Y : G T C T C T A G A

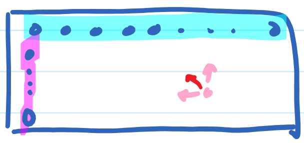
CCTAA is LCS of X, Y ; $X_{1..8}, Y$; $X_{1...7}, Y$

CCTA is LCS of $X_{1...6}, Y_{1...8}$

$$\text{lcs-len}(i,j) = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ 1+\text{lcs-len}(i-1,j-1) & \text{if } x_i = y_j \\ \max(\text{lcs-len}(i,j-1), \text{lcs-len}(i-1,j)) & \text{otherwise} \end{cases}$$

length of LCS of $X_{1...i}, Y_{1,...,j}$

$\Theta(n\cdot m)$ entries
$\Theta(1)$ to calc each
$\Theta(n\cdot m)$ total



THM: Let Z be LCS of X and Y

Then if $x_m = y_n$ then $Z_{1...k-1}$ is an LCS of $X_{1...m-1}, Y_{1...n-1}$

and if $x_m \neq y_n$ and $z_k \neq x_m$ then Z is an LCS of X_{m-1}, Y

and if $x_m \neq y_n$ and $z_k \neq y_n$ then Z is an LCS of X, Y_{n-1}

Proof : 1) Suppose Z is LCS of X, Y and $x_m = y_n$

Suppose further that $Z_{1...k-1}$ is not LCS of $X_{1...m-1}, Y_{1...n-1}$;
there is Z' s.t. Z' is LCS of $X_{1,...,m-1}, Y_{1,...,n-1}$
and $\text{len}(Z') > \text{len}(Z) - 1$

Then $Z' \cdot x_m$ is of X, Y

$\text{len}(Z' \cdot x_m) = \text{len}(Z') + 1 > \text{len}(Z)$
so Z not LCS

So $Z_{1...k-1}$ is LCS of ...

2) Suppose Z is LCS of X, Y, $z_k \neq x_m$ and $x_m \neq y_n$

Z is CS of $X_{1\ldots m-1}, Y$:

Z is <u>L</u>CS of $X_{1\ldots m-1}, Y$:

3) similar

LCS Example

		A	C	C	G	T	A	A	C	T
	0	0	0	0	0	0	0	0	0	0
G	0	0	0	0	1	1	1	1	1	1
T	0	0	0	0	1	2	2	2	2	2
C	0	0	1	1	1	2	2	2	3	3
T	0	0	1	1	1	2	2	2	3	4
C	0	0	1	2	2	2	2	2	3	4
T	0	0	1	2	2	3	3	3	3	4
A	0	1	1	2	2	3	4	4	4	4
G	0	1	1	2	3	3	4	4	4	4
A	0	1	1	2	3	3	4	5	5	5