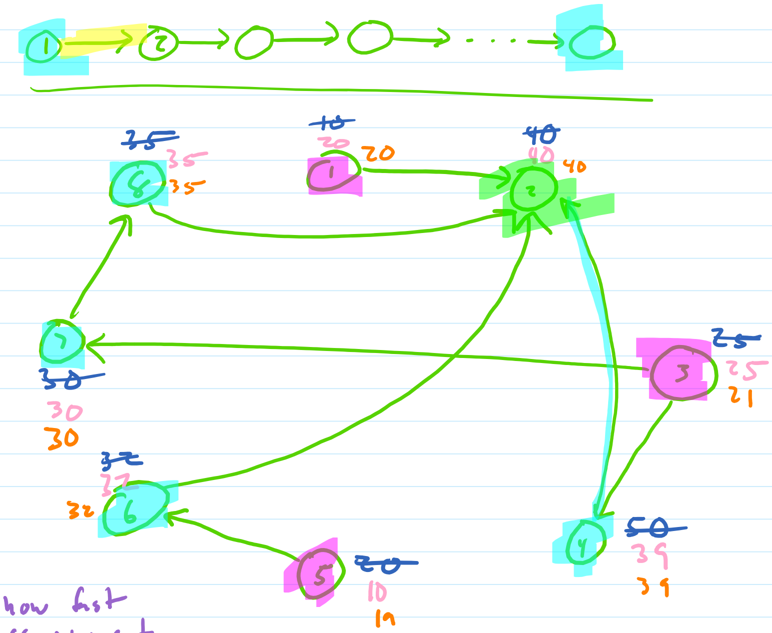
Adversary

MIN_AND_MAX (A, n)
    max ← A[1]
    min ← A[1]
    for i = 2 to n
        if A[i] > max
            max ← A[i]
        else if A[i] < min
            min ← A[i]
    return (min, max)

$2n-2$ comps



U unknown
X cand for max
N cand for min
O other

initially: all n in U

how fast can you get from init to end

end: 1 X, 1 N, n-2 O

U vs X
U > X: X/O
X > U: X/N

initially can move 2 from U in 1 comp by comparing U/U $\frac{n}{2}$

then X vs X to turn one X to O $\frac{n}{2}-1$
N vs N to turn one N to O $\frac{n}{2}-1$

$\frac{3}{2}n-2$

X N → X > N
X O → X > O
N O → O > N
O O → consistent w/ prev responses

need to get n-2 elts U → O (two steps $U \rightarrow X / N \rightarrow O$)
1 elt U → X
1 elt U → N

match!

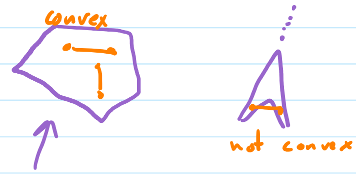
$2(n-2)+1+1$ steps total
$= 2n-2$

can do $\frac{n}{2}$ U vs U comps to do 2 steps / comp
but then no more U's, so remaining steps 1 / comp

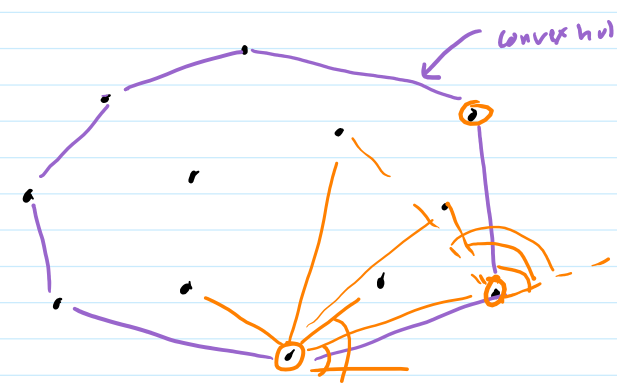
so total comps required is $\frac{n}{2}+(n-2) = \frac{3}{2}n-2$

Reductions

Convex Hull



Given a set of points in the plane, what convex polygon with vertices chosen from the set contains the rest?



$O(n^2)$

$O(n \cdot \underbrace{h}_{\text{\# pts on conv hull (output sensitive)}})$

Sorting w/ Convex Hull

CH-SORT($\overbrace{A}^{\text{list of real number}}$)

easy
- for each $x \in A$
  - pts ← pts ∪ { (x, $-x^2$) } $\Theta(n)$

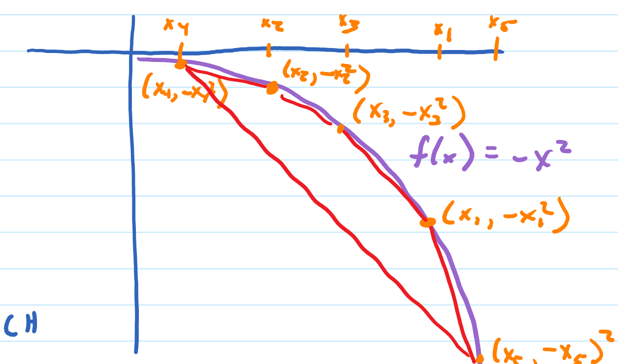
ch = CONVEX-HULL(pts) — better than $O(n \log n)$

easy

easy
- sort ← [ ]
- for each $(x, y) \in$ ch
  - sort ← sort + [x] $\Theta(n)$

return sort

$\Theta(n)$ + time for CH



convex hull = [$(x_4, -x_4^2)$, $(x_2, -x_2^2)$, ..., $(x_5, -x_5^2)$]

take x-coords: [x_4, x_2, x_3, x_1, x_5]

if could solve CH in time better than $O(n \log n)$
then we can sort in time better than $O(n \log n)$
but know we can't do this

so can't do CH in better than $O(n \log n)$
lower bound

reduction from $\underset{\text{sorting}}{A}$ to $\underset{\text{convex hull}}{B}$ shows if B is easy (low hardness) then A is easy

$a \le b$ = if A not easy then B is not easy
if a is high then b is high — if A is hard (high hardness) then B is hard

$A \le_p B$

SORT $\le_p$ CONVEX HULL

given undir G, is there a path that visits each vertex exactly once

HAMILTONIAN-PATH $\le_p$ LONGEST-PATH

↳ given undir G, find length of

$A =_p B$

SORT $\leq_p$ CONVEX HULL

HAMILTONIAN-PATH $\leq_p$ LONGEST-PATH

↳ given undir G, find length of longest simple path in G

Solving HP using LP:

HP(G)
    $n \leftarrow$ # verts in G] poly-time
    long $\leftarrow$ LONGEST-PATH(G)
    if long $= n-1$
        return TRUE
    else
        return FALSE] poly-time

if there is a poly-time alg for LP, then there is a poly-time alg for HP

if no poly-time alg for HP then no poly-time alg for LP

P : set of problems w/ poly-time algs that solve them

NP: a class of useful problems (non-deterministic poly-time)

$X \in NP$

NP-complete: the hardest of the problems in NP

X is NP-complete : 1) X is in NP
2) for all Y in NP, $Y \leq_p X$