Machinist Optimality

m	PrefM	w	PrefW		another stable matching
A	XYVWZ	V	ADCEB	~~(A,X)~~ (A,Y)	(A,V)
B	VXWYZ	W	ABDCE	~~(B,V)~~ (B,W)	(B,W)
C	VZWYX	X	DECAB	~~(C,V)~~ (C,Z)	(D,X)
D	WVXZY	Y	CBAED	~~(D,W)~~ (D,V)	(C,Y)
E	XYVWZ	Z	ABDEC	(E,X)	(E,Z)

w is a valid partner for m if exists stable matching M s.t. (m,w)

V, Y are valid partners for A

best(m) is m's best valid partner m's most preferred valid partner

Y = best(A)

G-S always returns $S = \{(m, best(m)) \mid m \in M\}$

m's preference list : $w_1, \ldots, w_i, \ldots, w_n$ (Invites; best(m))

INVARIANT i) $\forall m$, MatchM(m) $\neq$ NIL $\to$ MatchM(m) = $\min_m$ w over (m,w) $\in$ Invites

if have invited best valid then are matched w/ best valid

j) $\forall m$, (m, best(m)) $\in$ Invites $\to$ MatchM(m) = best(m)

so $\forall$m,w, (m,w) $\in$ Invites $\to$ w $\geq_m$ best(m)

Base : Invites = $\emptyset$ so all MatchM(m) = NIL for all m

Induction: Suppose invariant is true before loop and guard is also true

The only changes to Invites (and hence MatchM) are for

Case 1 : (m,w) only thing added to Invites
w = best(m) so :)
otherwise
w $\geq_m$ best(m)

Invites; best(m)
m: $w_1, \ldots, w, \ldots, w_n$

Case 2 a) Same as case 1 for m.

Suppose w = best(m') [will get to contradiction]

Find stable matching S s.t. (m',w) $\in$ M (w is valid partner of m')

Find w' s.t. (m, w') $\in$ S

w' is a valid partner of m

m $\in$ FreeM$_{old}$ (choice of m)

(m, w') $\notin$ Invites$_{old}$ (w' $\leq_m$ best(m) by def of best + w' valid; (m, best(m)) $\notin$ Invites$_{old}$ since MatchM$_{old}$(m) = NIL)

m prefers w to w' ✓

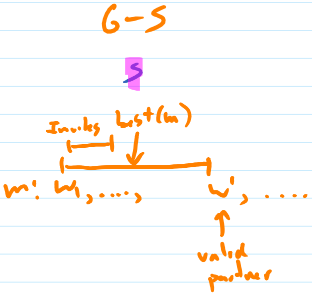
w prefers m to m' (how we got to 2a)

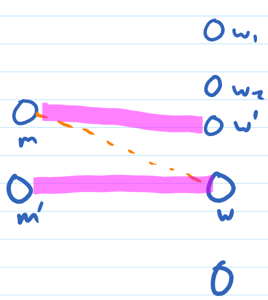
(m,w) instability in S ⇒⇐

wrecks Inv j

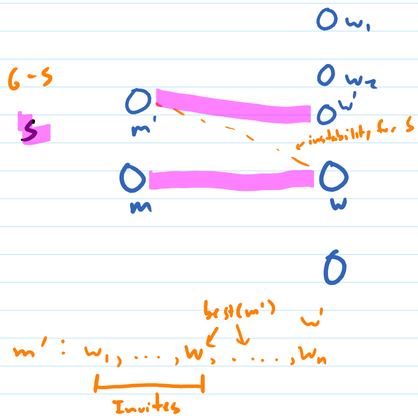
b) w stayed with m' ;

G-S





Suppose $w = best(m)$ [this would be bad - need a contradiction]



Find <u>stable matching S</u> s.t. $(m, w) \in S$ (w is (best) valid partner of m)

<u>w prefers m' to m</u> (case 2 b)

find w' s.t. $(m', w') \in S$ (S is a perfect matching)

so w' is valid partner of m' (def valid)

$MatchM(m') \neq w'$

$(m', w') \notin$ Invites (either $w' = best(m')$ and $MatchM(m') \neq w'$ or $w' <_{m'} best(m')$ but all invites to $w'' \geq_{m'} best(m')$)

$(m', w) \in$ Invites ($(m', w) \in$ Tent)

<u>m' prefers w to w'</u> (ordering of invitations)

(m', w) is an instability in S $\Rightarrow\Leftarrow$

Well Ordering Principle

(equivalent to induction)

Every non-empty set of nonnegative integers has smallest element

Equivalent to induction

(violates the invariant)

Suppose algorithm makes a mistake.

Then it makes a first mistake

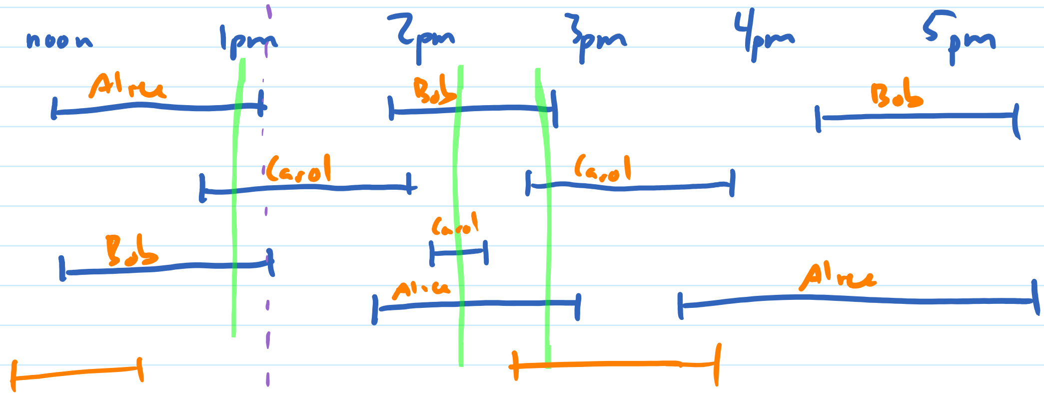
usually prove it must also have made earlier mistake

can't have mistake earlier than 1st $\Rightarrow\!\!\not\leftarrow$

or first mistake not a mistake (b/c no earlier mistakes) $\Rightarrow\!\!\not\leftarrow$

$\therefore$ no mistakes

Problem: Given set of intervals, find min people so that each has schedule of compatible intervals, and all intervals covered by someone



most overlapping = 3

so opt soln uses ≥ 3

simple greedy uses 3 people

∴ greedy is optimal

Free:

Alice	Bob	Carol
✗	✗	✗
✓	✓	✓
✗	✗	