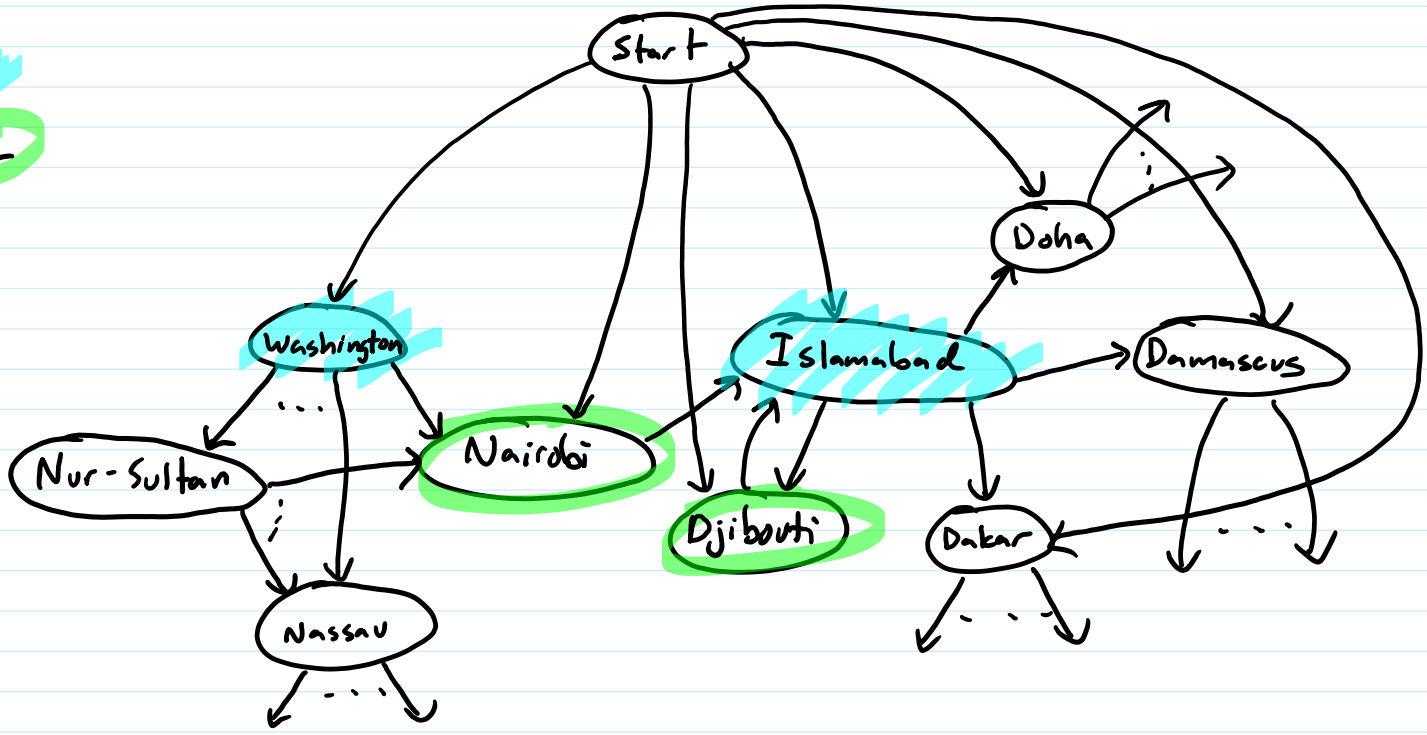
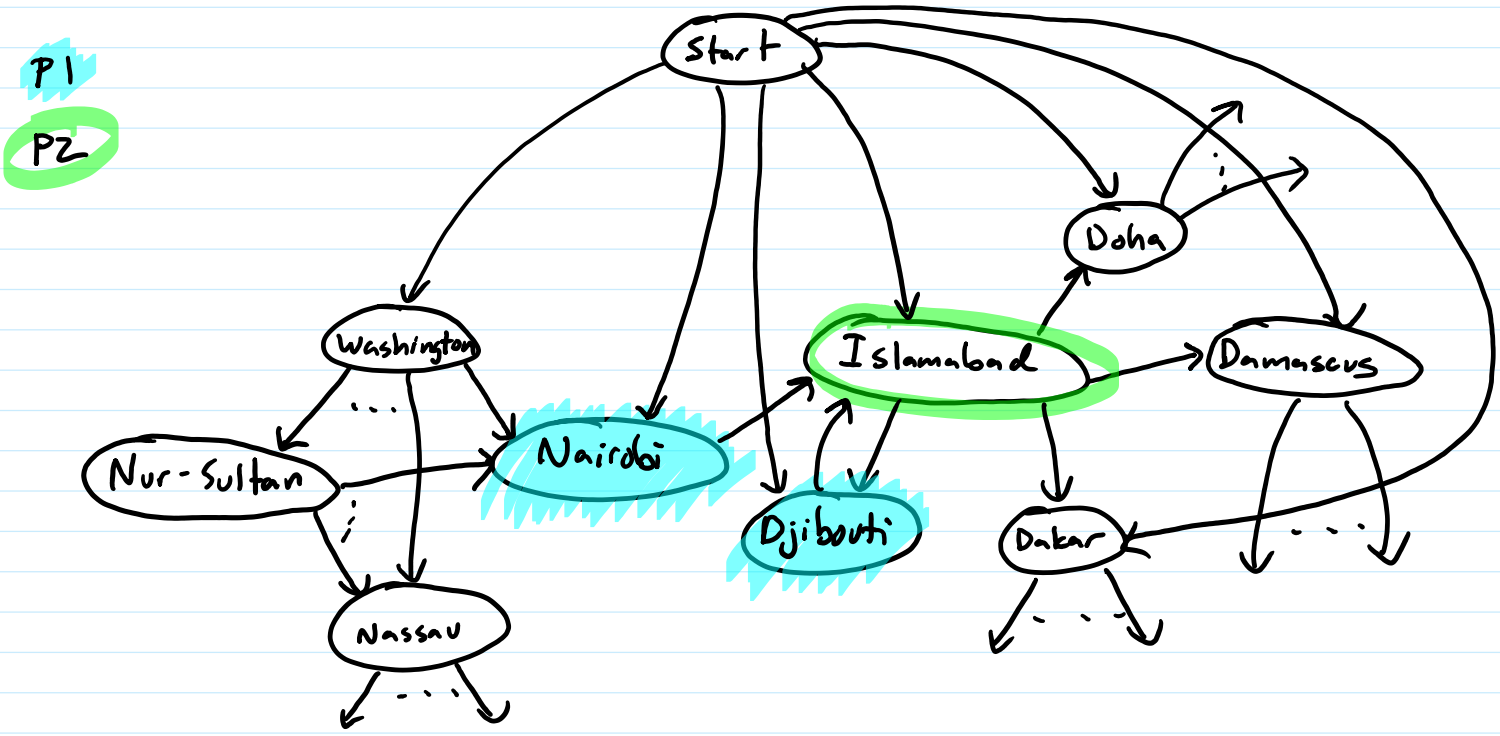
P1

P2



GEOGRAPHY: given a directed graph G with starting vertex, does player 1 have a winning strategy?



GEOGRAPHY: given a directed graph G with starting vertex, does player 1 have a winning strategy?

GEOGRAPHY ∈ PSPACE

```
GEOGRAPHY(G,s)
visited[v] = False for all v
return EVAL(G, s, 1, visited)
```

one call per branch of game tree
(exponential)

recursion depth < n
1 array of size n per recursive call
$O(n^2)$ space

```
EVAL(G,v,p,visited)
copy ← copy of visited with copy[v] = True
if p=1 then
   for all u adjacent to v
      if visited[u] = False
         if EVAL(G,u,2, copy) = True
            return True
   return True
else
   for all u adjacent to v
      if visited[u] = False
         if EVAL(G,u,1, copy) = False
            return False
   return True
```

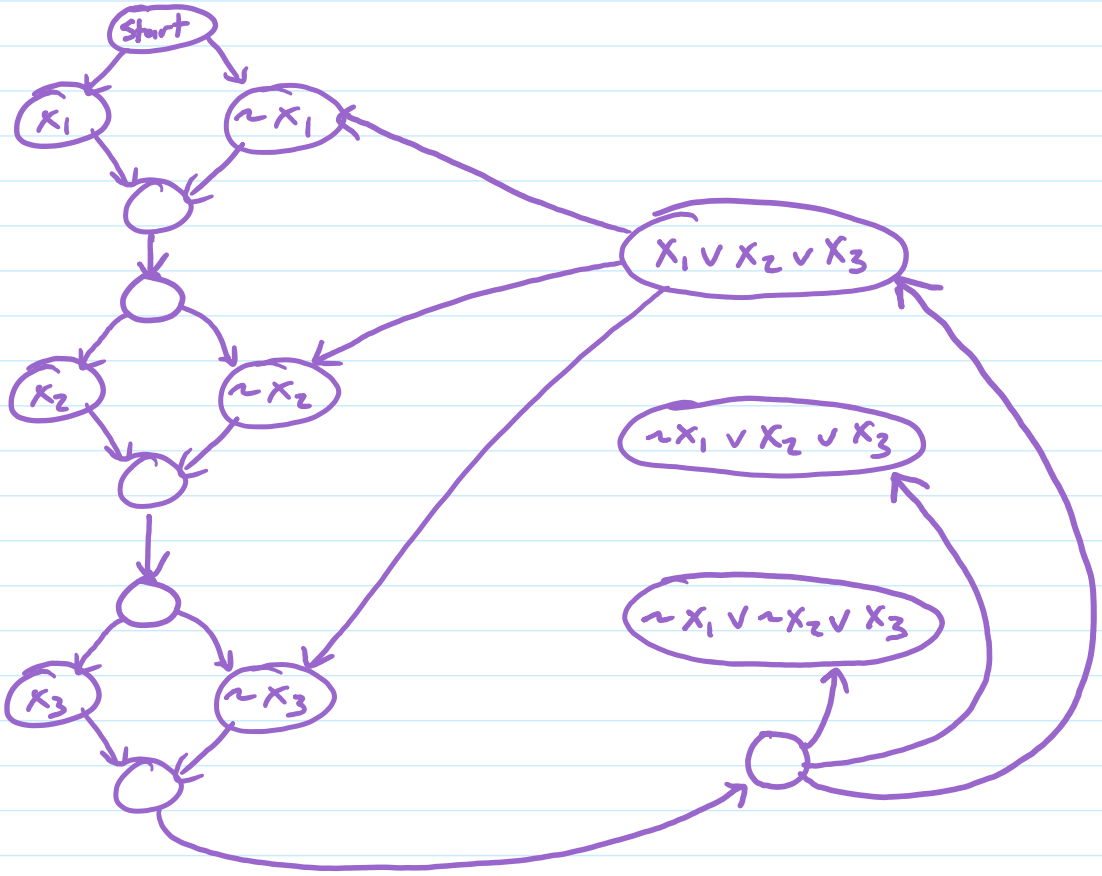
QSAT is PSPACE-complete

QSAT $\leq_p$ GEOGRAPHY

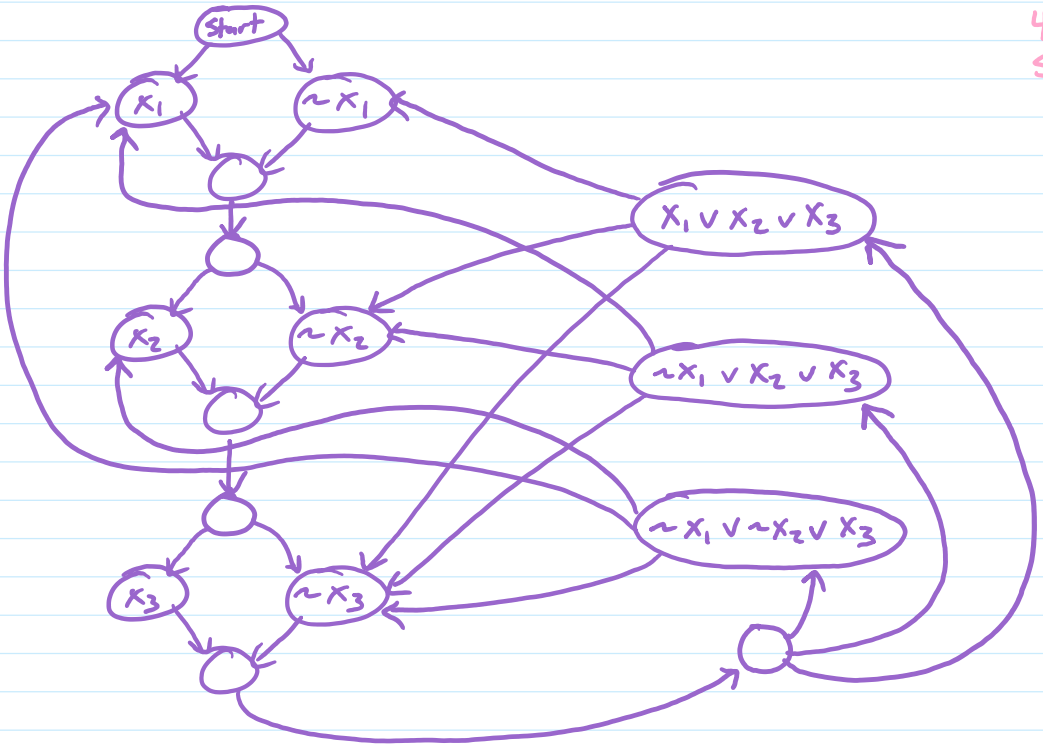
So GEOGRAPHY is PSPACE-complete

QSAT to GEOGRAPHY

$$(x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee \sim x_2 \vee x_3)$$



$$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$

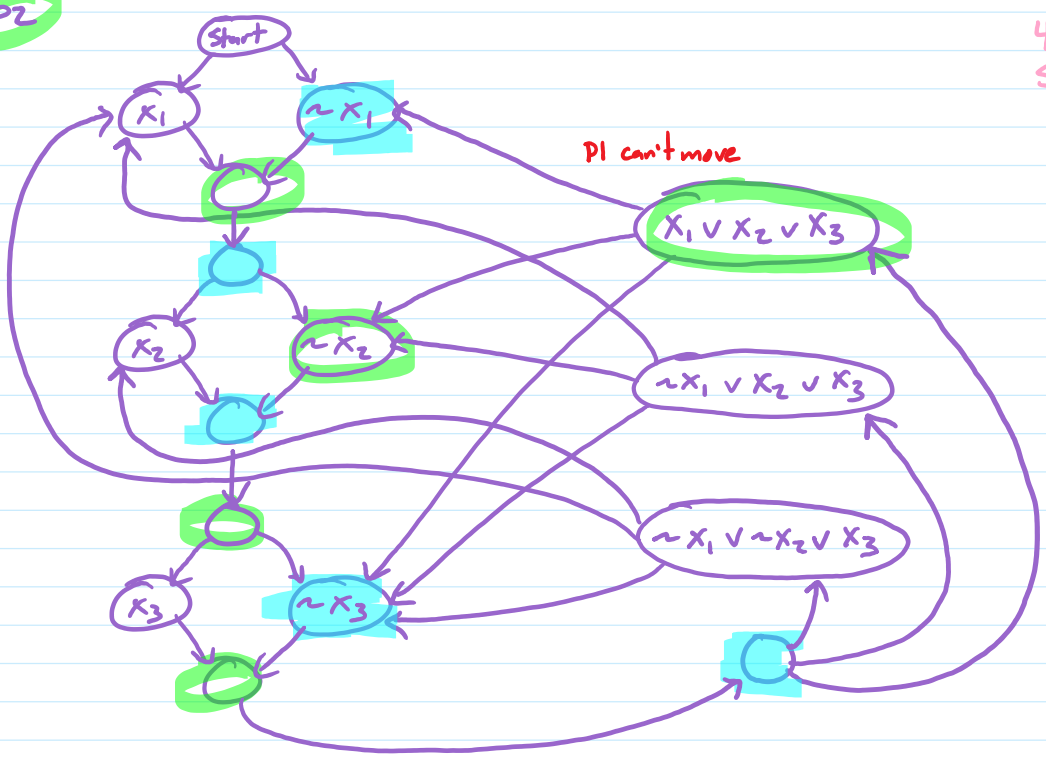


$4n + m + 1$ vertices
$\leq 5n + 4m$ edges

so polynomial space to build
and polynomial time

P1
P2

$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$



$4n + m + 1$ vertices
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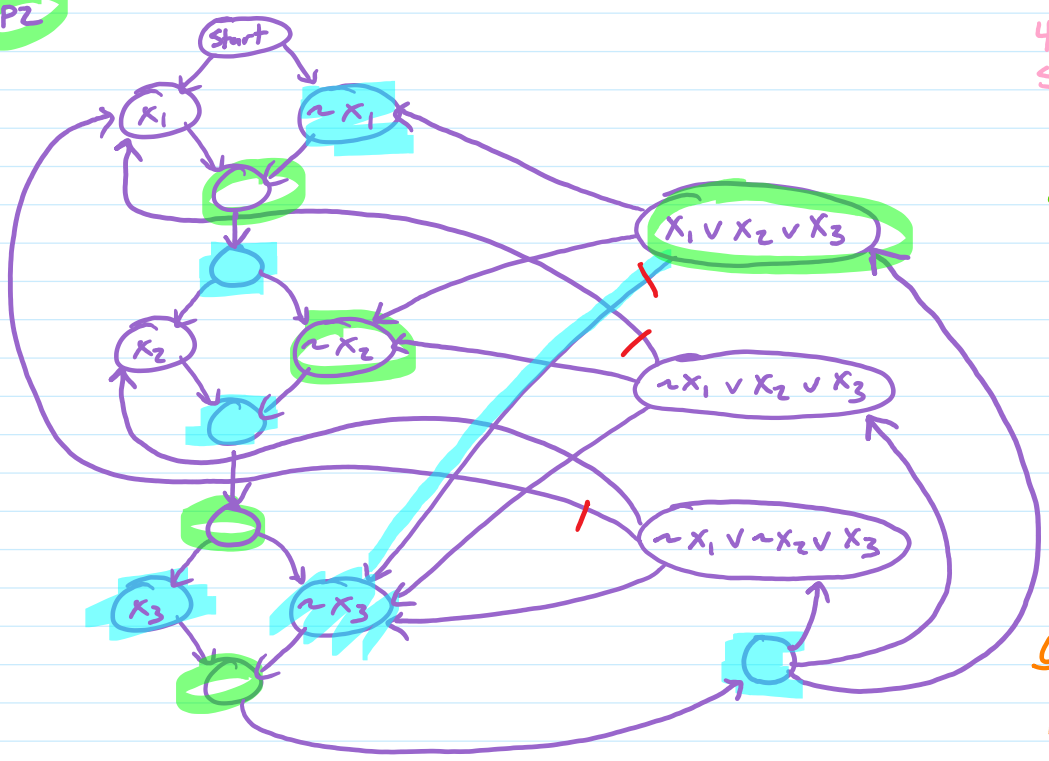
so polynomial space to build
and polynomial time

P1
P2

$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$



$4n + m + 1$ vertices
$\leq 5n + 4m$ edges

so polynomial space to build
and polynomial time

P1 has winning strategy
↕
Every clause has T term
↕
Formula was made T

QSAT(φ)
build G
return GEOGRAPHY(G, start)