Sequential Search

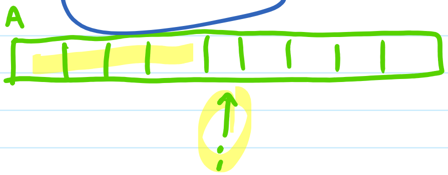
PRE: A is an array and elts are = comparable w/ key

POST: returns index i s.t. A[i]==key and $0 \le i < len(A)$ if key in array

-1 otherwise

```
find(A, key)
  i <- 0
  while i < len(A) and A[i] != key
    i <- i + 1
  if i < len(A)
    return i
  else
    return -1
```

1	2	1	2	2



Termination: when $n = len(A)$

$i = n = len(A)$

so condition is F and loop terminates

Post: 2 cases

a) $i < len(A)$

Now $A[i] = key$

→ so there is an occurrence of key so must return i s.t. $A[i] = key$ which we do

b) $i \ge len(A)$

$i \le len(A)$ from INV b)

$i = len(A)$

by c) $A[0], \dots, A[len(A)-1] \ne key$

all of A

key is not in A

return should be -1 and is

INV: a) $i = n$ and ✓

b) $0 \le i \le len(A)$ ✓

c) $A[0], \dots, A[i-1]$ all $\ne$ key

and i is an integer

↳ for all k s.t. $0 \le k \le i-1$, $A[k] \ne key$

$0 \le k \le -1$

Base case ($n=0$): i init to 0 so $i = n$

and $0 \le i \le len(A)$

and $A[0], \dots, A[i-1] \ne key$

so vacuously true

Ind: Suppose INV T after n iterations and $i_{old} < len(A)$ and $A[i_{old}] \ne key$

[want INV T w/ new values for n+1]

$i_{old} = n$ [want $i_{new} = n+1$]

$i_{new} = i_{old} + 1 = n+1$ ✓

$0 \le i_{old} \le len(A)$ and $i_{old} < len(A)$

so $0 \le i_{old} \le len(A) - 1$

$0 \le 1 \le i_{old} + 1 \le len(A) - 1 + 1$

and $0 \le i_{new} \le len(A)$

$A[0], \dots, A[i_{old} - 1] \ne key$

and $A[i_{old}] \ne key$

so $A[0], \dots, A[i_{old}] \ne key$

$A[0], \dots, A[i_{new} - 1] \ne key$

Binary Search

PRE: $A[0] \le A[1] \le \cdots \le A[\mathrm{len}(A)-1]$ (A is sorted)

POST: returns T if key is in array, F otherwise

```
int start = 0;
int end = len(A) - 1;
while (start <= end and A[(start + end) / 2] != key)
  {
    int mid = (start + end) / 2;
    if (key < a[mid]) {
        end = mid - 1;
      }
    else {
        start = mid + 1;
      }
  }

if (start > end or A[(start + end) / 2] != key)
 return false;
else
 return true;
```

INVARIANT

$key > A[start-1]$

$key < A[end+1]$

$0 \le start \le \mathrm{len}(A)$

$-1 \le end \le \mathrm{len}(A)$

$end - start + 1 \le \mathrm{len}(A) - n$

(or $\le \frac{\mathrm{len}(A)}{2^n}$)