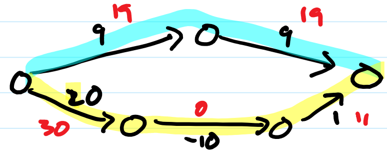
Negative Weights

Negative weights



If $s, v_1, v_2, \ldots, v_k$ is a shortest path $s \rightsquigarrow v_k$ then

shortest path $w \le n-1$ edges

for each v_i, v_j $\quad v_i, v_{i+1}, \ldots, v_j$ is shortest path $v_i \rightsquigarrow v_j$

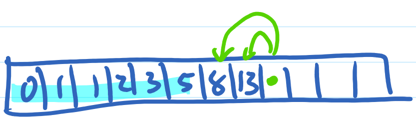
shortest path $s \rightsquigarrow v_{k-1}$ w/ $\le n-2$

~~$d(v)$ = total weight of shortest path $s \rightsquigarrow v$~~

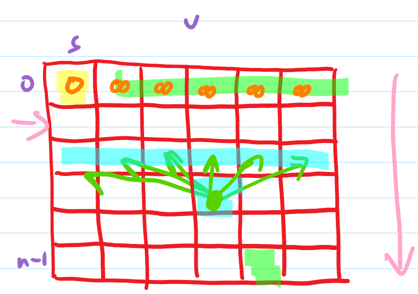
$$\cancel{= \begin{cases} 0 & \text{if } v = s \\ \min\limits_{(u,v) \text{ is an edge}} d(u) + w(u,v) & \text{otherwise} \end{cases}}$$

Let $d_s(i, v)$ = tot weight of short path $s \rightsquigarrow v$ w/ $\le i$ edges

$$= \begin{cases} 0 & \text{if } v = s \\ \infty & \text{if } i = 0 \text{ and } v \ne s \\ \min\left(\min\limits_{(u,v) \text{ is an edge}} d_s(i-1, u) + w(u,v)\right), d(i-1, v)) & \text{otherwise} \end{cases}$$



$\Theta(n^2)$ space



SHORTEST-PATHS(n, s)

$M \leftarrow n \times n$ array
for $v = 0$ to $n-1$ $\quad M[0, v] \leftarrow \infty$
$M[0, s] \leftarrow 0$

$\Theta(n^2)$ iterations
for $i = 1$ to $n-1$
  for $v = 0$ to $n-1$
    $M[i, v] = \min(M[i-1, v], \underbrace{\min\limits_{u=0 \ldots n-1} M[i-1, u] + w(u, v)}_{\Theta(n) \text{ per iteration}})$

$\Theta(n^3)$ total time

SHORTEST-PATHS(n, s)

```
M ← n×n array
for v=0 to n-1   M[~~0~~, v] ← ∞
M[~~0~~, s] ← 0

for i=1 to n-1
    for v=0 to n-1
        M[~~i~~, v] = min( M[~~i-1~~, v], min_{u=0...n-1} M[~~i-1~~, u] + w(u,v) )
```

SHORTEST-PATHS(n, s)

$\Theta(n^3)$ time

$\Theta(n)$ space

```
M ← n×n array
for v=0 to n-1   M[v] ← ∞
M[s] ← 0

for i=1 to n-1
    for v=0 to n-1
        M[v] = min( M[v], min_{u=0...n-1} M[u] + w(u,v) )
```

after iteration i: M[v] = length of a path s⇝v

≤ shortest path s⇝v using at most i edges

SHORTEST-PATHS(n, s)
BELLMAN-FORD

$\Theta(n \cdot m)$ time

$\Theta(n)$ space

```
M ← n×n array
for v=0 to n-1   M[v] ← ∞
M[s] ← 0

for i=1 to n-1
    for all edges (u,v)
        M[v] = min( M[v], M[u] + w(u,v) )
```

1-2-3 Nim

Start with one rows of n stones

On each turn, take as ~~1, 2, or 3~~ 1,3,4 stones one row

If no possible moves, you lose (last move wins)

win(n) = T if player to make next move has winning strategy for n stones

$$= \begin{cases} F & \text{if } n=0 \\ \neg win(n-1) \vee \neg win(n-3) \vee \neg win(n-4) & \text{otherwise} \end{cases}$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
F	T	F	T	T	T	T	F	T	F	T	T	T	T	F		

Vendor Scheduling

Week

	1	2	3	4	5
Brooklyn	6	3	10	4	1
Stratford	4	6	3	8	20

23

21

$M = 10$

net profit 41

$6+3+10+8+20-10 = 37$

1st 4 weeks of optimal schedule for 5 weeks $\neq$ optimal schedule for 4 weeks

where is the optimal substructure?