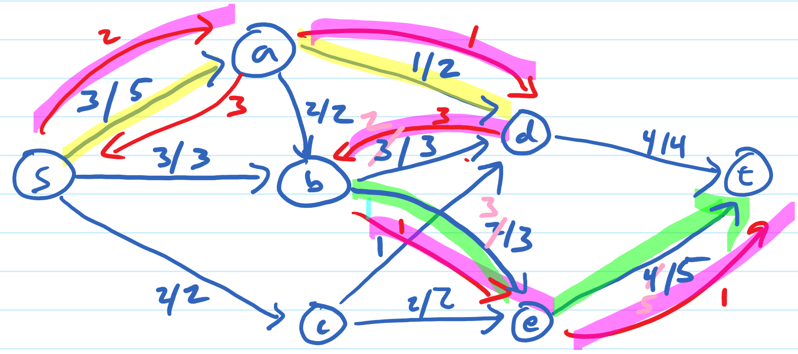
**Backward Edges**




F-F $O(m \cdot C)$

total capacity from source

pseudopolynomial

Pseudopolynomial

Thursday, April 2, 2020 2:34 PM

COUNT-DIVISORS(13)

1101

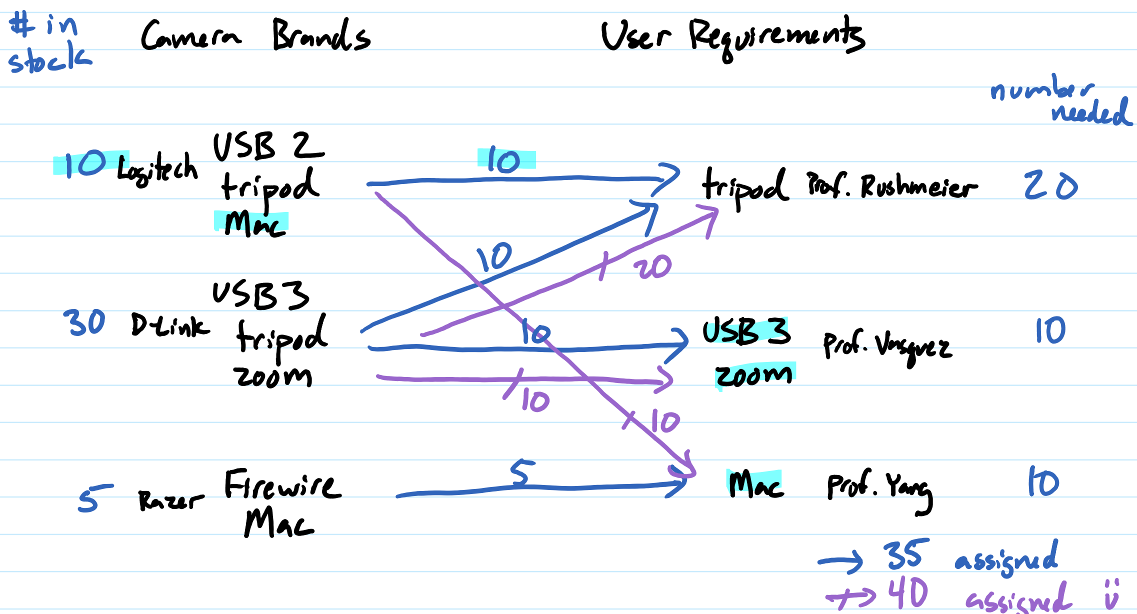
4 bits

```
COUNT-DIVISORS(n)
  count ← 0
  for i = 1 to n          ← O(n) iterations
      if n % i == 0
          count ← count + 1     ] O(1)
  return count
```

O(n) total

↪ $O(2^{\#\text{ bits input}})$ exponential

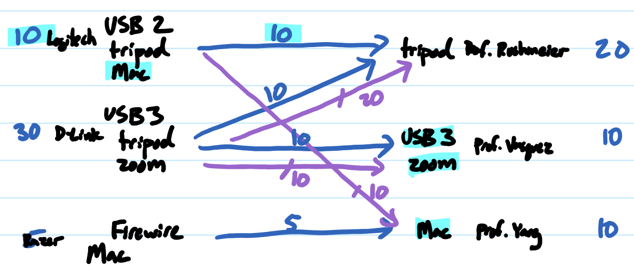
pseudopolynomial!




Goal: maximize # cameras sent out

subject to constraints

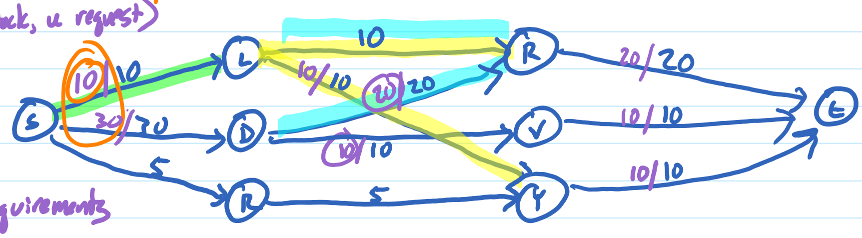
1) can't assign more of one brand than in stock
2) can't assign more to user than requested
3) can't assign unless have required capabilities




or anything greater

construct G with
$c(b,u) = \min(b \text{ in stock}, u \text{ request})$
$c(s,b) = b$ in stock
$c(u,t) = u$'s request

only have an edge if brand b satisfies u's requirements




$\rightarrow \; f^{out}(L) = f^{in}(L) = f(s,L) \le c(s,L) = 10$

In general, for any flow, $f^{out}(b) \le \#\ b$ available

$\rightarrow \; f^{in}(V) = f^{out}(V) = f(V,t) \le c(V,t) = 10$

$f^{in}(u) \le \#u$ requested

$\#$ cameras distribute $= \sum_b \#$ brand b distributed

$= \sum_b \sum_u \#$ brand b distributed to user u

#cameras distributed

=

$$= \sum_b \sum_u \text{\# brand } b \text{ distributed } b \text{ user } u$$

$$= \sum_b \underline{\sum_u f(b,u)}$$

$$= \sum_b f^{out}(b)$$

$$= \sum_b f^{in}(b) = \sum_b f(s,b) = \underline{v(f)}$$

#cameras distributed
=
value of flow

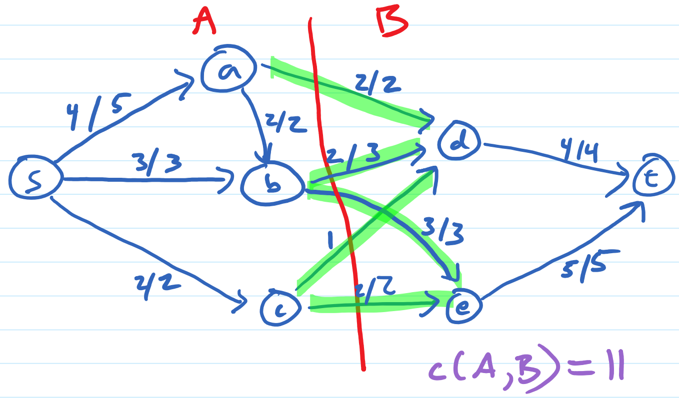
so max flow gives
max distribution

(solve with Scaling
Ford-Fulkerson)

# Cuts

s-t cut : partition $V$ into $S$, $V-S$

$s \in S$ $\qquad$ $t \in V-S$

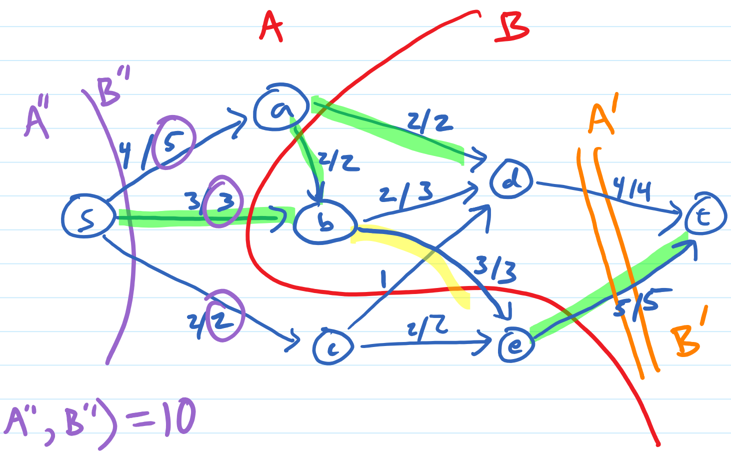



$c(A,B) = 11$

$$f(A,B) = \sum_{\substack{(a,b)\in E \\ a\in A, b\in B}} f(a,b) - \sum_{\substack{(a,b)\in E \\ a\in B, b\in A}} f(a,b)$$

$$= 9 - 0$$

$$= \underset{\sim}{9} = v(f)$$




$c(A'',B'') = 10$

$f(A,B) = 12 - 3 = \underset{\sim}{9} = v(f)$

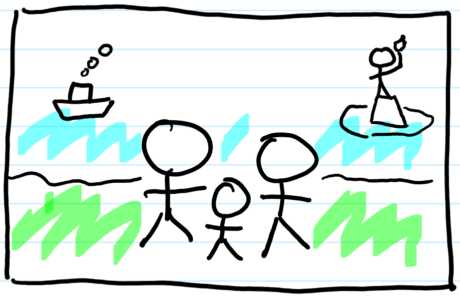
$c(A,B) = 13$

$f(A',B') = 9 = v(f)$

$\underline{c(A',B')} = \underset{\sim}{9}$

$$c(A,B) = \sum_{\substack{(u,v)\in E \\ u\in A, v\in B}} c(u,v)$$

## Background Segmentation



Determine what pixels are background, which are foreground.

For each, give $a_i = \underline{P(\text{pixel } i \text{ in fore})}$ $\quad b_i = P(\text{pixel } i \text{ in back})$ $\quad a_i + b_i = 1$

consult image processing experts for how to get these – measure blur?

For neighboring pairs, give $p_{ij}$ = penalty for separating pixel $i$ from $j$ (one in fore, one in back)

Find partition $\overset{\text{fore}}{A}, \overset{\text{back}}{B}$ to maximize $\displaystyle\sum_{i \in A} a_i + \sum_{i \in B} b_i - \sum_{\substack{i,j \text{ adjacent} \\ i \in A \text{ xor } j \in A}} p_{ij}$

$$= \sum_{i \in A} (1 - b_i) + \sum_{i \in B} (1 - a_i) - \sum_{\substack{i,j \text{ adjacent} \\ i \in A \text{ xor } j \in A}} p_{ij}$$

$$= |A| - \sum_{i \in A} b_i + |B| - \sum_{i \in B} a_i - \sum_{\substack{i,j \text{ adjacent} \\ i \in A \text{ xor } j \in A}} p_{ij}$$

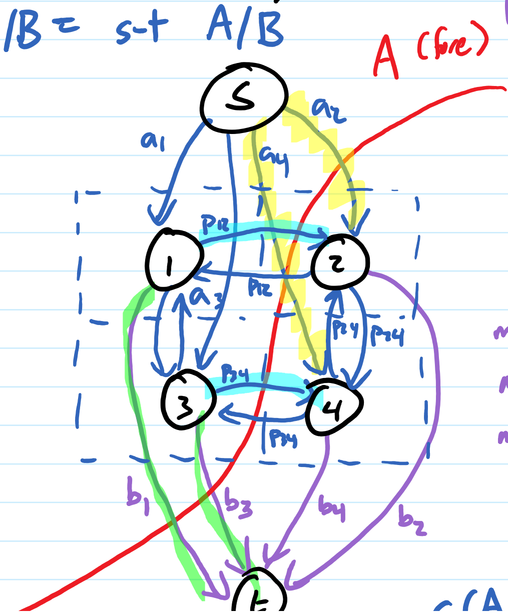
$$= \underbrace{Q}_{\text{total \# pixels}} - \sum_{i \in A} b_i - \sum_{i \in B} a_i - \sum_{\substack{i,j \text{ adjacent} \\ i \in A \text{ xor } j \in A}} p_{ij}$$

minimize $\displaystyle\sum_{i \in A} b_i + \sum_{i \in B} a_i + \sum_{\substack{i,j \text{ adjacent} \\ i \in A \text{ xor } j \in A}} p_{ij}$

$= c(A, B)$

partition $A/B$ = s-t $A/B$

3 || 2 || 4 || 3




min cut
=
min subtrahend
=
max score

want to find $A, B$ to minimize capacity of s-t cut $(A, B)$

$\Downarrow$

Ford-Fulkerson (max-flow $\Leftrightarrow$ min cut)

$c(A,B) = \sum b_i + \sum a_i + \sum p_{ij}$


$$c(A,B) = \sum_{i\in A} b_i + \sum_{i\in B} a_i + \sum_{\substack{i,j \text{ adjacent} \\ i\in A,\, j\in B \\ \text{or} \\ i\in B,\, j\in A}} p_{ij}$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| PrefM | $w_5$ | $w_7$ | $w_1$ | $w_3$ | $w_2$ | ~~$w_4$~~ | ~~$w_0$~~ | ~~$w_6$~~ |
| PrefW | $m_3$ | $m_7$ | $m_4$ | $m_2$ | $m_1$ | $m_0$ | $m_5$ | $m_6$ |

Match $(m_3, w_5)$ $(m_7, w_7)$ $(m_4, w_0)$ $(m_2, w_2)$ $(m_1, w_4)$

$(m_0, w_1)$ $(m_5, w_2)$ $(m_6, w_3)$