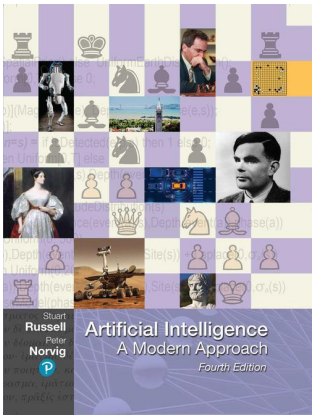
Artificial Intelligence: A Modern Approach

Fourth Edition



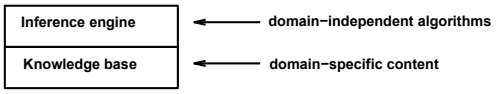
Chapter 7

Logical agents

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - resolution
  - forward chaining
  - backward chaining
- Effective Propositional Model Checking

Knowledge bases



Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):
Tell it what it needs to know
Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level
i.e., data structures in KB and algorithms that manipulate them

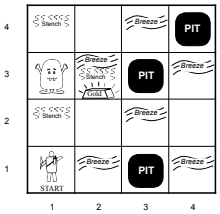
A simple knowledge-based agent

```
function KB-Agent( percept) returns an action
  static: KB, a knowledge base
          t, a counter, initially 0, indicating time
  Tell(KB, Make-Percept-Sentence( percept, t))
  action ← Ask(KB, Make-Action-Query(t))
  Tell(KB, Make-Action-Sentence(action, t))
  t ← t + 1
  return action
```

The agent must be able to:
Represent states, actions, etc. Incorporate new percepts
Update internal representations of the world Deduce hidden properties of the world Deduce appropriate actions

Wumpus World PEAS description

Performance measure
gold +1000, death -1000
-1 per step, -10 for using the arrow Environment
Squares adjacent to wumpus are smelly Squares adjacent to pit are breezy Glitter iff gold is in the same square
Shooting kills wumpus if you are facing it Shooting uses up the only arrow
Grabbing picks up gold if in same square
Releasing drops the gold in same square



Actuators Left turn, Right turn, Forward, Grab, Release, Shoot
Sensors Breeze, Glitter, Smell

Wumpus world characterization

Observable?
?

Wumpus world characterization

Observable?? No—only local perception

Deterministic??

Wumpus world characterization

Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly

specified Episodic??

Wumpus world characterization

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level of actions Static??

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do not move Discrete??

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Single-agent??

Wumpus world characterization

Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly

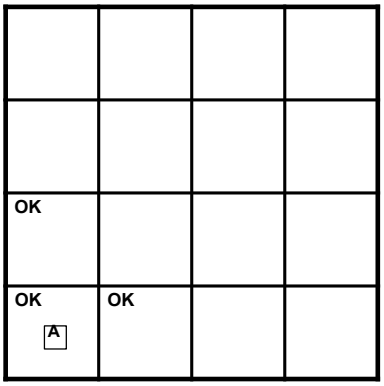
specified Episodic?? No—sequential at the

level of actions Static?? Yes—Wumpus and Pits

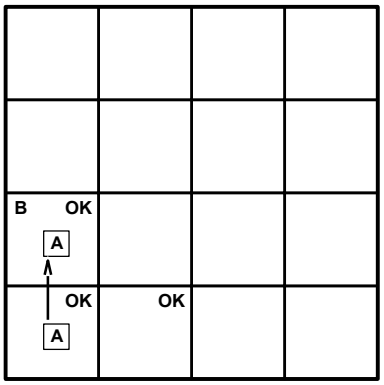
do not move Discrete?? Yes

Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



Exploring a wumpus world



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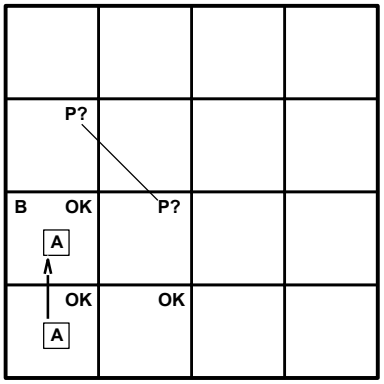

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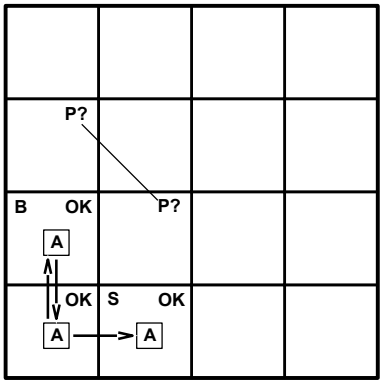

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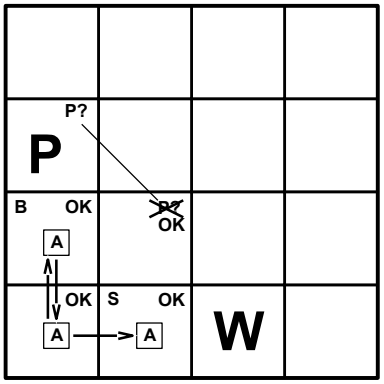

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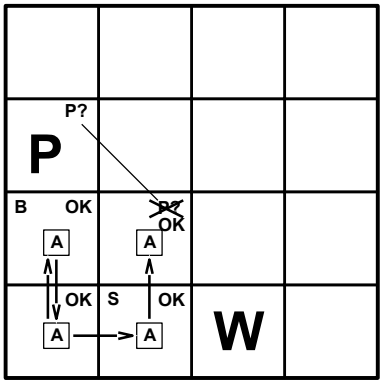

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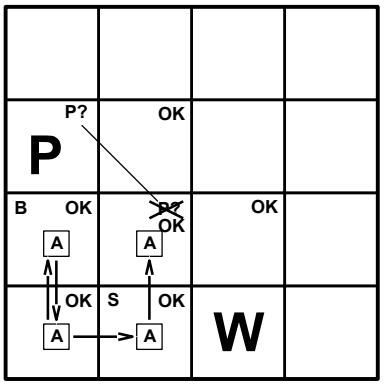

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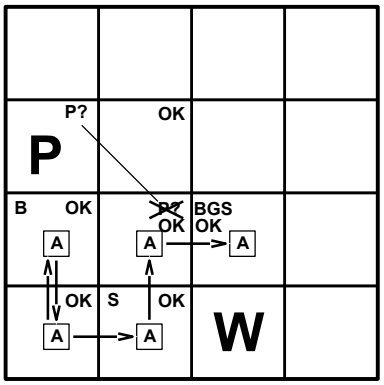
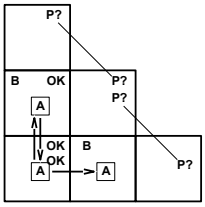
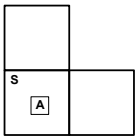


Other tight spots



Breeze in (1,2) and (2,1)
⇒ no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1)
⇒ cannot move
Can use a strategy of
coercion: shoot straight ahead ⇒ safe
wumpus was there ⇒ dead wumpus wasn't there
⇒ safe

Logic in general

Logics are formal languages for representing information such that conclusions can be drawn
Syntax defines the sentences in the language

Semantics define the "meaning" of

sentences;

i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence
$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number

$y \quad x + 2 \geq y$ is true in a world where $x = 7, \quad y = 1$

$x + 2 \geq y$ is false in a world where $x = 0, \quad y = 6$

Entailment

Entailment means that one thing follows from another:

$KB \models \alpha$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Note: brains process syntax (of some sort)

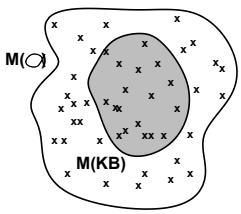
Models

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

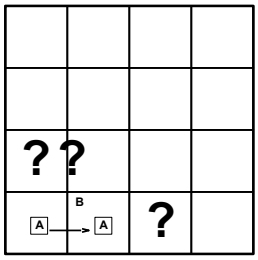
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. KB = Giants won and Reds won
α = Giants won



Entailment in the wumpus world

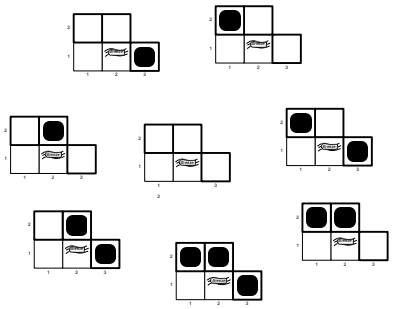
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]



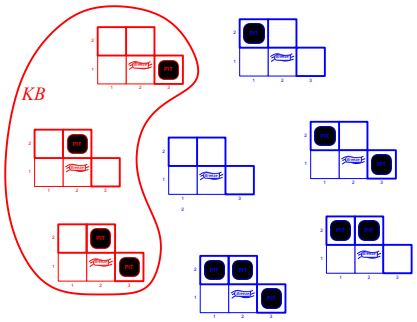
Consider possible models for ?s assuming only pits

3 Boolean choices ⇒ 8 possible models

Wumpus models

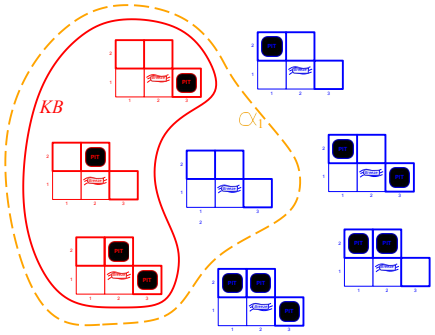


Wumpus models



KB = wumpus-world rules + observations

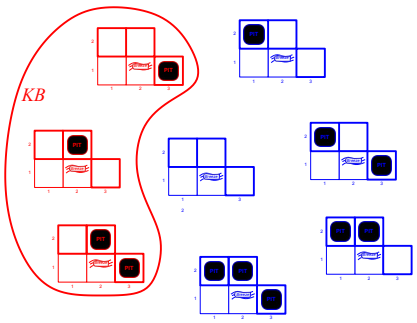
Wumpus models



KB = wumpus-world rules + observations

a_1 = "[1,2] is safe", $KB \models a_1$, proved by model checking

Wumpus models



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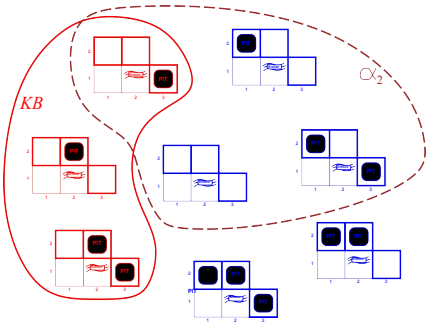
Wumpus models



KB = wumpus-world rules + observations

a_2 = "[2,2] is safe", $KB \models a_2$

Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas
The proposition symbols P_1, P_2 etc are

sentences If S is a sentence, $\neg S$ is a sentence

(negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence

(conjunction) If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a

sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
true *true* *false*

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

$\neg S$ is true iff S is false
$S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true
$S_1 \vee S_2$ is true iff S_1 is true or S_2 is true
$S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true
i.e., is false iff S_1 is true and S_2 is false
$S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,
$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$\neg P_{1,1}$
$\neg B_{1,1}$
$B_{2,1}$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$\neg P_{1,1}$
$\neg B_{1,1}$
$B_{2,1}$

"Pits cause breezes in adjacent squares"
$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
"A square is breezy if and only if there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
.	.	.	.	.	.	.	.	.	.	.	.	.
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
fals	tru	fals	fals	fals	tru	fals	tru	tru	tru	tru	tru	tru
e	e	e	e	e	e	e	e	e	e	e	e	e
fals	tru	fals	fals	fals	tru	tru	tru	tru	tru	tru	tru	tru
e	e	e	e	e	e	e	e	e	e	e	e	e
false	true	false	false	true	false	false	true	false	false	true	true	false
.	.	.	.	.	.	.	.	.	.	.	.	.
true	true	true	true	true	true	true	false	true	true	false	true	false

Enumerate rows (different assignments to symbols), if KB is true in row, check that α is too

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, α) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
          α, the query, a sentence in propositional logic
  symbols ← a list of the proposition symbols in KB and α
  return TT-Check-All(KB, α, symbols, [ ])

function TT-Check-All(KB, α, symbols, model) returns true or false
  if Empty?(symbols) then
      if PL-True?(KB, model) then return PL-True?(α, model)
      else return true
  else do
      P ← First(symbols); rest ← Rest(symbols)
      return TT-Check-All(KB, α, rest, Extend(P , true, model ))
      and
             TT-Check-All(KB, α, rest, Extend(P , false, model ))
```

$O(2^n)$ for n symbols; problem is co-NP-complete

Logical equivalence

Two sentences are logically equivalent iff true in same models:
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of $\vee$
$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$ implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of $\vee$ over $\wedge$

Validity and satisfiability

A sentence is valid if it is true in all models,
e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:
$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model
e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:
$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable
i.e., prove α by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules
- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
  Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking
truth table enumeration (always exponential in n)
improved backtracking, e.g.,
Davis–Putnam–Logemann–Loveland heuristic search in model space (sound but incomplete)
e.g., min-conflicts-like hill-climbing algorithms

Resolution

Conjunctive Normal Form (CNF—universal)
conjunction of disjunctions of literals
cla uses
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

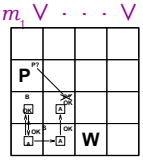
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \quad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals.
E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3} \vee P_{2,3}}$$

Resolution is sound and complete for propositional logic



Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$
2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$
3. Move $\neg$ inwards using de Morgan's rules and double-negation:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$
4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

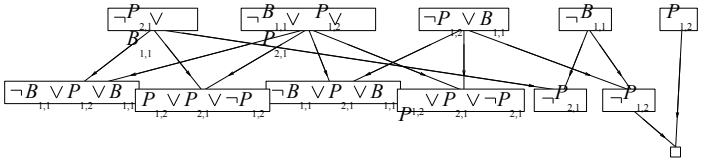
Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-Resolution(KB, α) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
          α, the query, a sentence in propositional logic
  clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
  new ← { }
  loop do
    for each C_i, C_j in clauses do
      resolvents ← PL-Resolve(C_i, C_j)
      if resolvents contains the empty clause then return true
      new ← new ∪ resolvents
    if new ⊆ clauses then return false
    clauses ← clauses ∪ new
```


Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$



Forward and backward chaining

Horn Form (restricted)

KB = conjunction of Horn clauses

Horn clause =

- proposition symbol; or
- (conjunction of symbols) $\Rightarrow$ symbol E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

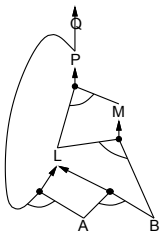
$$\frac{\alpha_1, \ldots, \alpha_n, \quad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining. These algorithms are very natural and run in linear time

Forward chaining

Idea: fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found

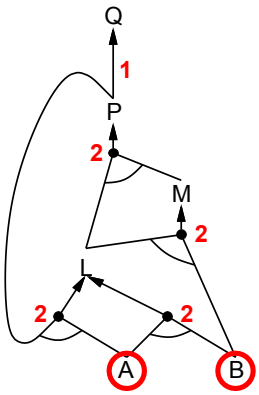
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B



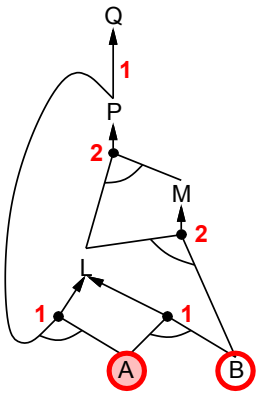
Forward chaining algorithm

```
function PL-FC-Entails?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
          q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in KB
  while agenda is not empty
    do p ← Pop(agenda)
    unless inferred[p] do
      inferred[p] ← true
      for each Horn clause c in whose premise p appears do
        decrement count[c]
        if count[c] = 0 then do
          if Head[c] = q then return true
          Push(Head[c], agenda)
  return false
```

Forward chaining example



Forward chaining example



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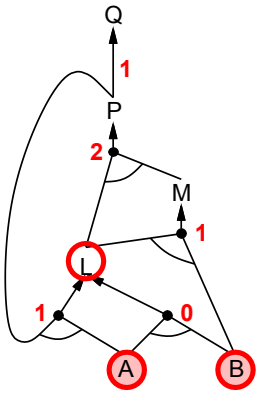

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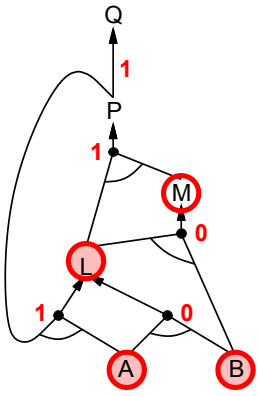

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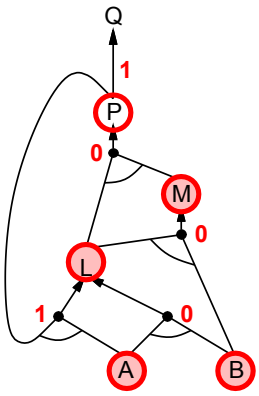

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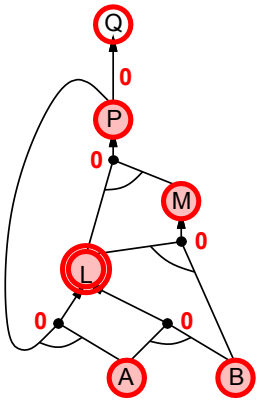

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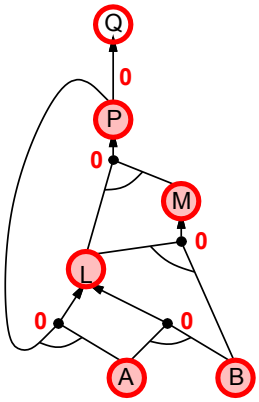

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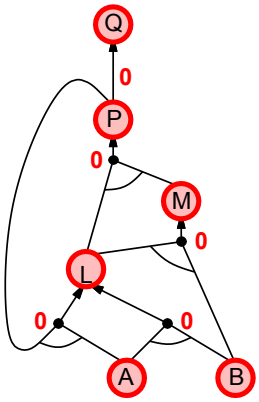


Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model m, assigning true/false to symbols
3. Every clause in the original KB is true in m
   Proof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in m Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m Therefore the algorithm has not reached a fixed point!
4. Hence m is a model of KB
5. If $KB \models q$, q is true in every model of KB, including m

General idea: construct any model of KB by sound inference, check a

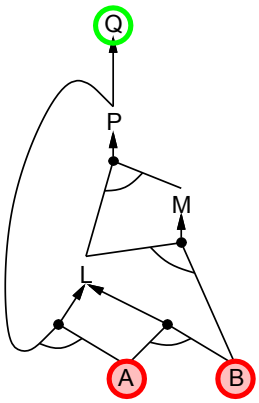
Backward chaining

Idea: work backwards from the query
q: to prove q by BC,
check if q is known already, or
prove by BC all premises of some rule concluding q
Avoid loops: check if new subgoal is already on the goal

stack Avoid repeated work: check if new subgoal

1) has already been proved true, or
2) has already failed

Backward chaining example



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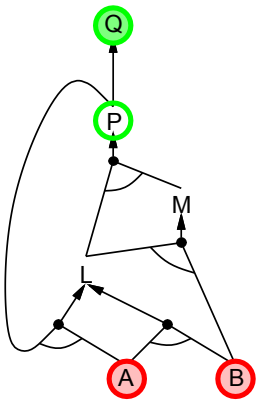

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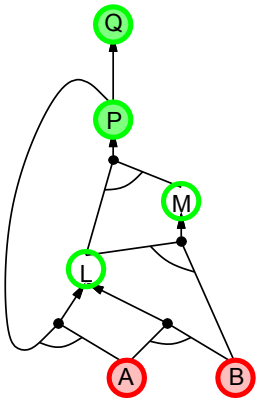

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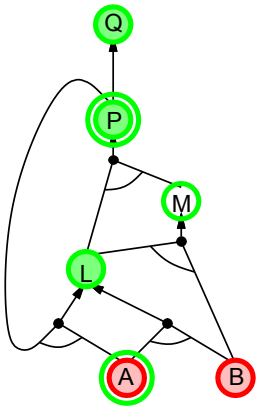

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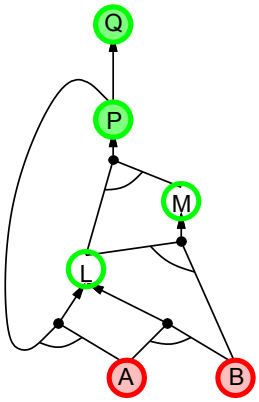

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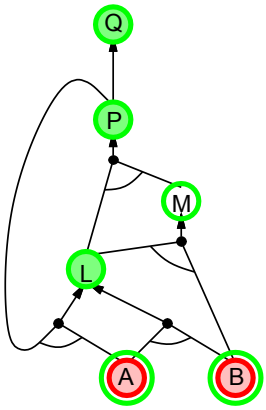

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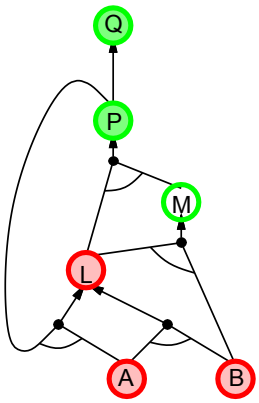

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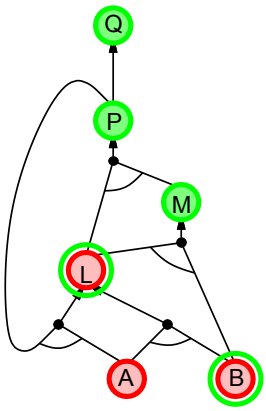

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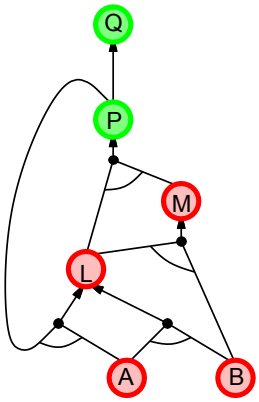

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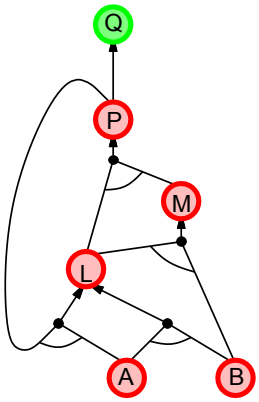

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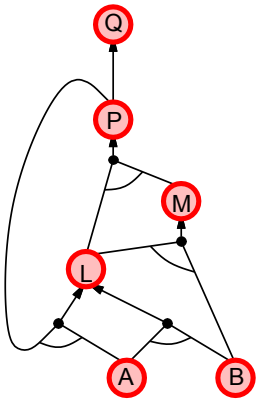

Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for

problem-solving,

e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB

Effective Propositional Model Checking

Davis–Putnam algorithm with three improvements over TT-ENTAILS

- Early termination: detect T/F
- Pure symbol heuristic: same sign in all clauses
- Unit Clause heuristic: clause with one literal

Local search algorithms such as hill-climbing & simulated annealing.

In recent years, there has been a great deal of experimentation to find a good balance between greediness and randomness.

WALKSAT: On every iteration, the algorithm picks an unsatisfied clause and picks a symbol in the clause to flip.

Summary

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated informa- tion, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses Resolution is complete for propositional logic. Propositional logic lacks expressive power

Local search methods (WALKSAT) find solutions (sound but not complete).