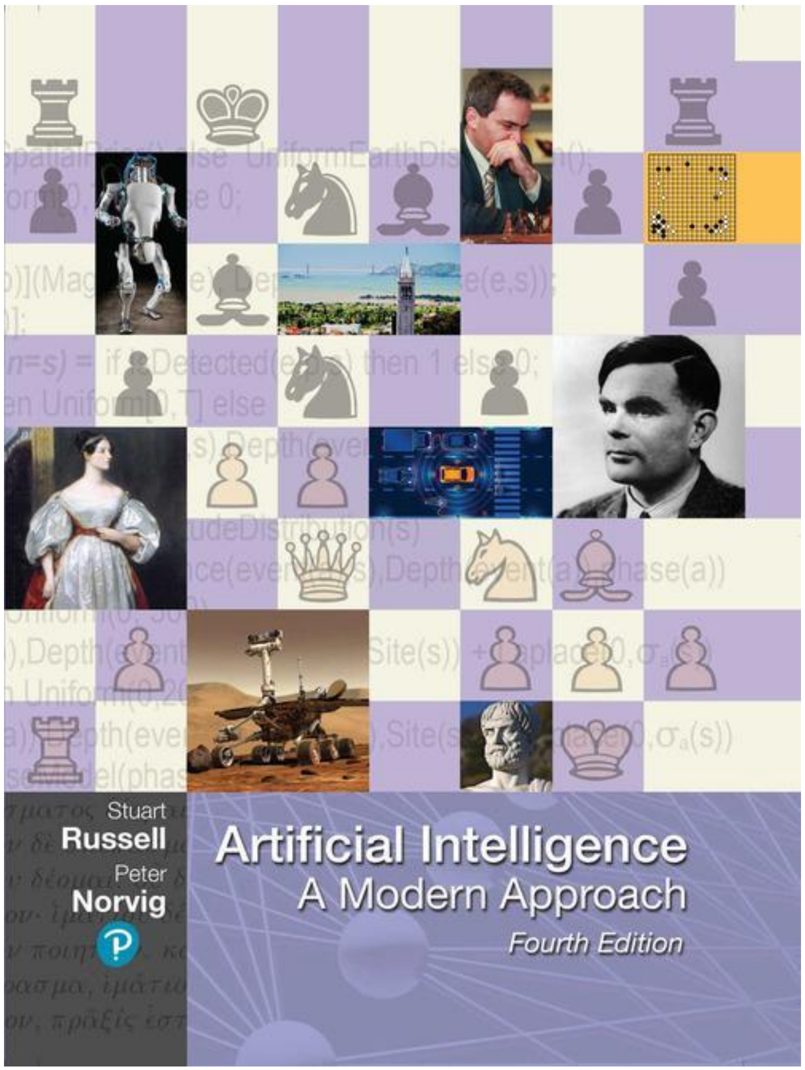
Artificial Intelligence: A Modern Approach

Fourth Edition



Chapter 8

First-order logic

Outline

- Why FOL?
- Syntax and semantics of FOL
- Fun with sentences
- Wumpus world in FOL
- Knowledge Engineering in FOL

Pros and cons of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases)
- Propositional logic is compositional: meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent (unlike natural language, where meaning depends on context)
- Propositional logic has very limited expressive power (unlike natural language)
  E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- **Objects**: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- **Relations**: red, round, bogus, prime, multistoried . . ., brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .
- **Functions**: father of, best friend, third inning of, one more than, end of
  . . .

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	*KingJ ohn*, **2**, *UCB*, .
Predicates	. .
Functions	*Brother*, >, . . .
Variables	*Sqrt*, *LeftLegOf*, . .
Connectiv	. *x*, *y*, *a*, *b*, . . .
es	$\wedge$ $\vee$ $\neg$ $\Rightarrow$ $\Leftrightarrow$
Equality	=
Quantifier	$\forall$ $\exists$
s	

Atomic sentences

Atomic sentence = $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term = $function(term_1, \ldots, term_n)$
or *constant* or *variable*

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$\neg S, \; S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \; S_1 \Leftrightarrow S_2$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$

$>(1, 2) \vee \leq(1, 2)$

$>(1, 2) \wedge \neg >(1, 2)$

Truth in first-order logic

Sentences are true with respect to a model and an interpretation
Model contains ≥ 1 objects (domain elements) and relations

among them Interpretation specifies referents for

constant symbols →

objects predicate symbols

→ relations

function symbols → functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$
is true iff the objects referred to by $term_1, \ldots,$
$term_n$
are in the relation referred to by *predicate*

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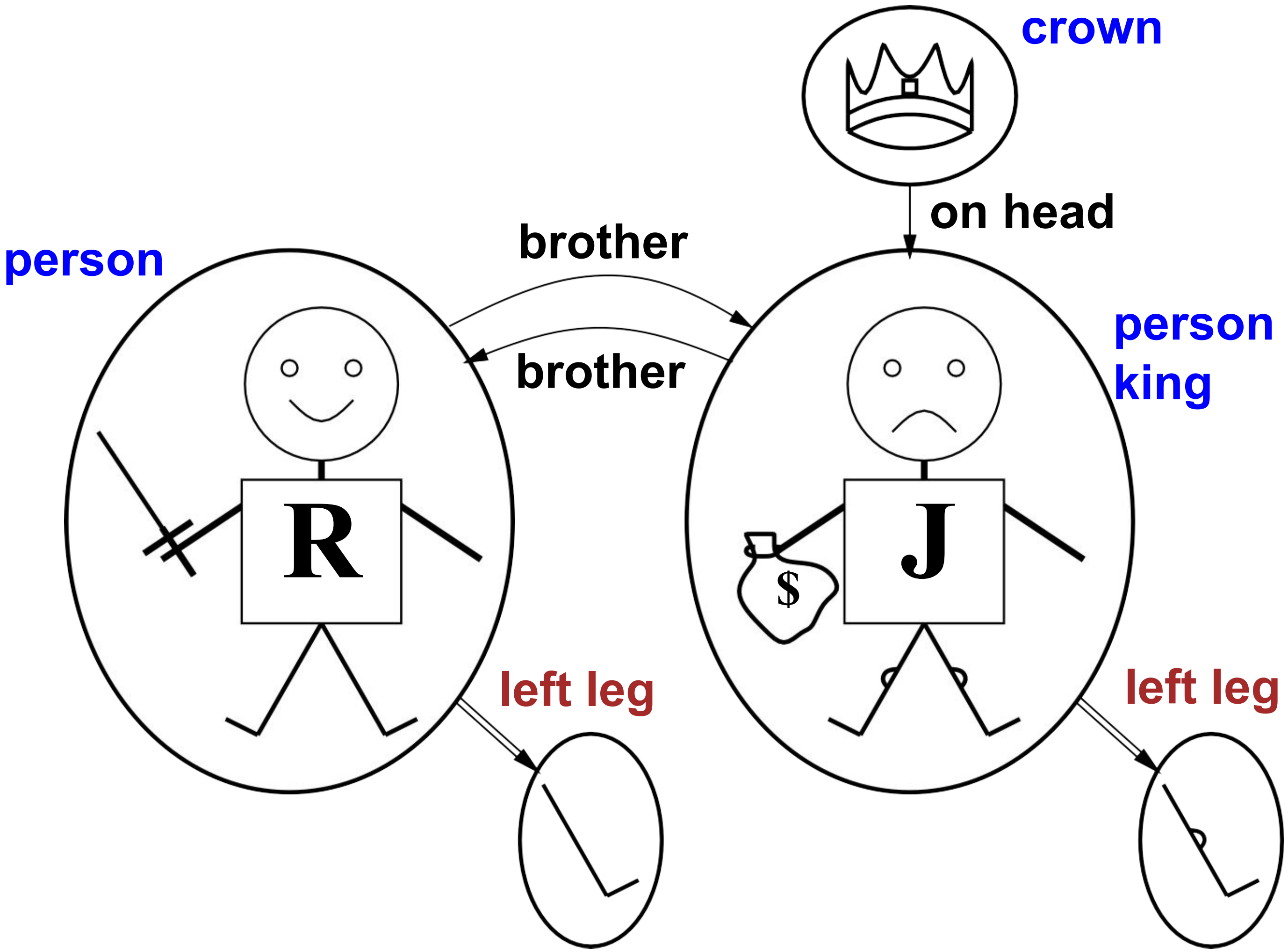
Models for FOL: Example



Truth example

Consider the interpretation in which $Richard \rightarrow$ Richard the Lionheart $John \rightarrow$ the evil King John
$Brother \rightarrow$ the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true just in case Richard the Lionheart and the evil King John are in the brotherhood relation in the model

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models
We can enumerate the FOL models for a given KB

vocabulary: For each number of domain elements n

from 1 to ∞

For each k-ary predicate P_k in the vocabulary For each possible k-ary relation on n objects

For each constant symbol C in the vocabulary

For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Universal quantification

$\forall$ }*variables* }*sentences*

Everyone at Berkeley is smart:
$\forall\, x \quad At(x, Berkeley) \quad \Rightarrow \quad Smart(x)$

$\forall\, x \quad P$ is true in a model m iff P is true with x being each possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of P

$$
\begin{aligned}
&(At(KingJohn, Berkeley) \Rightarrow Smart(KingJohn))\\
\wedge\ &(At(Richard, Berkeley) \Rightarrow Smart(Richard))\\
\wedge\ &(At(Berkeley, Berkeley) \Rightarrow Smart(Berkeley))\\
\wedge\ &\ldots
\end{aligned}
$$

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall\, x \;\; At(x, Berkeley) \wedge Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists$ }*variabless* }*sentences*

Someone at Stanford is smart:

$\exists\, x \quad At(x, Stanford) \wedge Smart(x)$

$\exists\, x \quad P$ is true in a model m iff P is true with x being some possible object in the model

Roughly speaking, equivalent to the disjunction of instantiations of P

$$
\begin{aligned}
&(At(KingJohn, Stanford) \wedge Smart(KingJohn)) \\
\vee\ &(At(Richard, Stanford) \wedge Smart(Richard)) \\
\vee\ &(At(Stanford, Stanford) \wedge Smart(Stanford)) \\
\vee\ &\ldots
\end{aligned}
$$

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Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists\, x\ \ At(x, Stanford) \Rightarrow Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall x\ \forall y$ is the same as $\forall y\ \forall x$ (why??)

$\exists x\ \exists y$ is the same as $\exists y\ \exists x$ (why??)

$\exists x\ \forall y$ is **not** the same as $\forall y\ \exists x$

$\exists x\ \forall y\ \ Loves(x, y)$

"There is a person who loves everyone in the world"

$\forall y\ \exists x\ \ Loves(x, y)$

"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x\ \ Likes(x, IceCream) \qquad \neg\exists x\ \neg Likes(x, IceCream)$

$\exists x\ \ Likes(x, Broccoli) \qquad \neg\forall x\ \neg Likes(x, Broccoli)$

Fun with sentences

Brothers are siblings

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$\forall\ x, y\ \ Brother(x, y)\ \ \Rightarrow$

$Sibling(x, y)$. "Sibling" is symmetric

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Brothers are siblings

$\forall\ x, y\ \ Brother(x, y) \quad \Rightarrow$

$Sibling(x, y)$. "Sibling" is symmetric

$\forall\ x, y\ \ Sibling(x, y) \quad \Leftrightarrow$

$Sibling(y, x)$. One's mother is

one's female parent

Fun with sentences

Brothers are siblings

$\forall\ x, y\ \ Brother(x, y) \quad \Rightarrow$

$Sibling(x, y)$. "Sibling" is symmetric

$\forall\ x, y\ \ Sibling(x, y) \quad \Leftrightarrow$

$Sibling(y, x)$. One's mother is

one's female parent

$\forall\ x, y\ \ Mother(x, y) \quad \Leftrightarrow \quad (Female(x) \land$

$Parent(x, y))$. A first cousin is a child of a parent's

sibling

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Fun with sentences

Brothers are siblings

$\forall\ x, y\ \ Brother(x, y) \Rightarrow$

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$\forall\ x, y\ \ Mother(x, y) \Leftrightarrow (Female(x) \wedge$

$Parent(x, y))$. A first cousin is a child of a parent's

sibling

$\forall\ x, y\ \ FirstCousin(x, y) \Leftrightarrow \exists\ p, ps\quad Parent(p, x) \wedge$
$Sibling(ps, p) \wedge$
$Parent(ps, y)$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall\, x \;\; \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid

E.g., definition of (full) *Sibling* in terms of *Parent*:

$$\forall\, x, y \;\; Sibling(x, y) \;\; \Leftrightarrow \;\; [\neg(x = y) \land \exists\, m, f \; \neg(m = f) \land Parent(m, x) \land Parent(f, x) \land Parent(m, y) \land Parent(f, y)]$$

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$
$Ask(KB, \exists\ a\ \ Action(a, 5))$
I.e., does KB entail any particular actions at $t = 5$?

Answer: $Yes, \{a/Shoot\}$ ← substitution (binding

list) Given a sentence S and a substitution σ,

$S\sigma$ denotes the result of plugging σ into S; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Knowledge base for the wumpus world

"Perception"

$\forall\ b, g, t \quad Percept([Smell, b, g], t) \ \Rightarrow\ Smelt(t)$

$\forall\ s, b, t \quad Percept([s, b, Glitter], t) \ \Rightarrow\ AtGold(t)$

Reflex: $\forall\ t\ AtGold(t) \ \Rightarrow\ Action(Grab, t)$

Reflex with internal state: do we have the gold already?

$\forall\ t\ AtGold(t) \wedge \neg Holding(Gold, t) \Rightarrow\ Action(Grab, t)$

$Holding(Gold, t)$ cannot be observed

$\Rightarrow$ keeping track of change is essential

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Deducing hidden properties

Properties of locations:

$\forall\, x, t \;\; At(Agent, x, t) \wedge Smelt(t) \;\Rightarrow\; Smelly(x)$

$\forall\, x, t \;\; At(Agent, x, t) \wedge Breeze(t) \Rightarrow\; Breezy(x)$

Squares are breezy near a pit:

Diagnostic rule—infer cause from effect

$$\forall\, y \;\; Breezy(y) \;\Rightarrow\; \exists\, x \quad Pit(x) \wedge Adjacent(x, y)$$

Causal rule—infer effect from cause

$$\forall\, x, y \quad Pit(x) \wedge Adjacent(x, y) \Rightarrow\; Breezy(y)$$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

Definition for the *Breezy* predicate:

$$\forall\, y \;\; Breezy(y) \;\Leftrightarrow\; [\exists\, x \quad Pit(x) \wedge Adjacent(x, y)]$$

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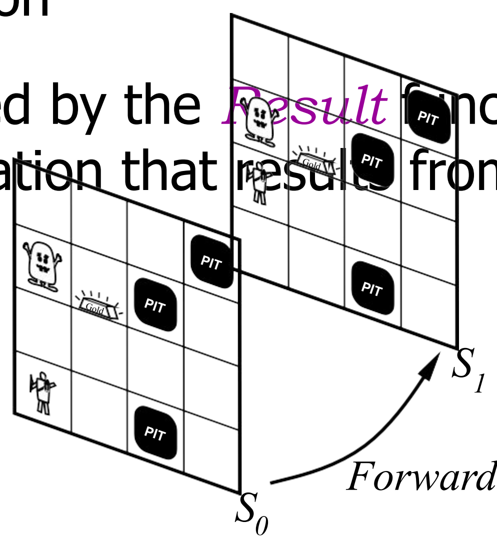
Keeping track of change

Facts hold in situations, rather than eternally
E.g., *Holding*(*Gold, Now*) rather than just *Holding*(*Gold*)

Situation calculus is one way to represent change in FOL:

Adds a situation argument to each non-eternal predicate E.g., *Now* in *Holding*(*Gold, Now*) denotes a situation

Situations are connected by the *Result* function
Result(*a, s*) is the situation that results from doing *a* in *s*



Describing actions I

"Effect" axiom—describe changes due to action

$\forall\, s \quad AtGold(s) \;\Rightarrow\; Holding(Gold, Result(Grab, s))$

"Frame" axiom—describe non-changes due to action

$\forall\, s \quad HaveArrow(s) \Rightarrow\; HaveArrow(Result(Grab, s))$

Frame problem: find an elegant way to handle non-change

(a) representation—avoid frame axioms

(b) inference—avoid repeated "copy-overs" to keep track of state

Qualification problem: true descriptions of real actions require endless caveats— what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences— what about the dust on the gold, wear and tear on gloves, . . .

Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is "about" a predicate (not an action per se):

P true afterwards $\Leftrightarrow$ [an action made P true

$\vee$ P true already and no action made P false]

For holding the gold:

$$\forall\, a, s \;\; Holding(Gold, Result(a, s)) \Leftrightarrow [(a = Grab \wedge AtGold(s)) \vee (Holding(Gold, s) \wedge a = Release)]$$

Making plans

Initial condition in KB:

$At(Agent, [1, 1], S_0)$
$At(Gold, [1, 2], S_0)$

Query: $Ask(KB, \exists\ s\ \ Holding(Gold, s))$
i.e., in what situation will I be holding the gold?

Answer: $\{s/Result(Grab, Result(Forward, S_0))\}$
i.e., go forward and then grab the gold

This assumes that the agent is interested in plans starting at S_0 and that S_0
is the only situation described in the KB

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Making plans: A better way

Represent plans as action sequences $[a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing p in s

Then the query $Ask(KB, \exists\, p\;\; Holding(Gold, PlanResult(p, S_0)))$ has the solution $\{p/[Forward, Grab]\}$

Definition of $PlanResult$ in terms of $Result$:

$\forall\, s \quad PlanResult([\,], s) = s$

$\forall\, a, p, s \quad PlanResult([a|p], s) = PlanResult(p, Result(a, s))$

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner

Knowledge Engineering in FOL

Knowledge engineering: the general process of knowledge-base construction.

The steps used in the knowledge engineering process:

1. Identify the questions.
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the problem instance
6. Pose queries to the inference procedure and get answers
7. Debug and evaluate the knowledge base

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Knowledge Engineering in FOL

Applications in the electronic circuits domain

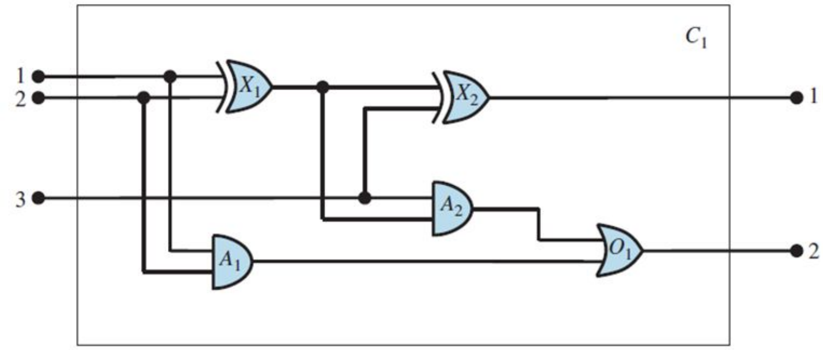


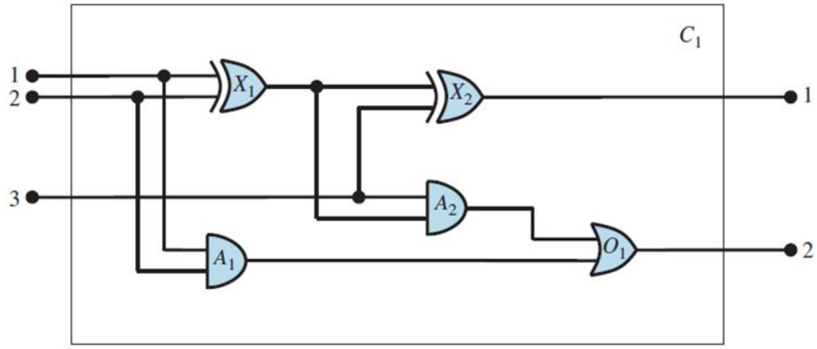
Figure 8.6 A digital circuit C_1, purporting to be a one-bit full adder. The first two inputs are the two bits to be added, and the third input is a carry bit. The first output is the sum, and the second output is a carry bit for the next adder. The circuit contains two XOR gates, two AND gates, and one OR gate.

1. Identify the questions

- Does the circuit in Figure 8.6 actually add properly?
- If all the inputs are high, what is the output of gate A2?
- Questions about the circuit's structure are also interesting.
- For example, what are all the gates connected to the first input terminal?
- Does the circuit contain feedback loops?

Knowledge Engineering in FOL

Applications in the electronic circuits domain



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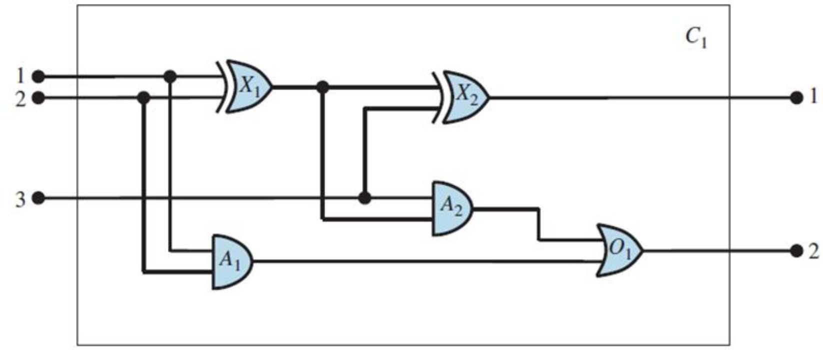
2. Assemble the relevant knowledge

- Circuits composed of wires and gates.
- Signals flow along wires to the input terminals of gates
- Each gate produces a signal on the output terminal that flows along another wire.
- There are four types of gates: AND, OR, and XOR gates have two input terminals, and NOT gates have one.

Knowledge Engineering in FOL

Applications in the electronic circuits domain



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3. Decide on a vocabulary

- Each gate is represented as an object named by a constant, about which we assert that it is a gate with
- $Gate(X_1)$, eg: $Type(X_1) = XOR$
- $Circuit(C_1)$
- $Terminal(x)$

Knowledge Engineering in FOL

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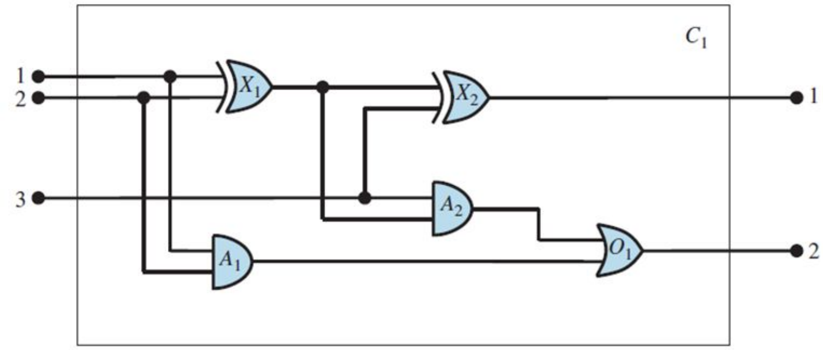

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4. Encode general knowledge of the domain

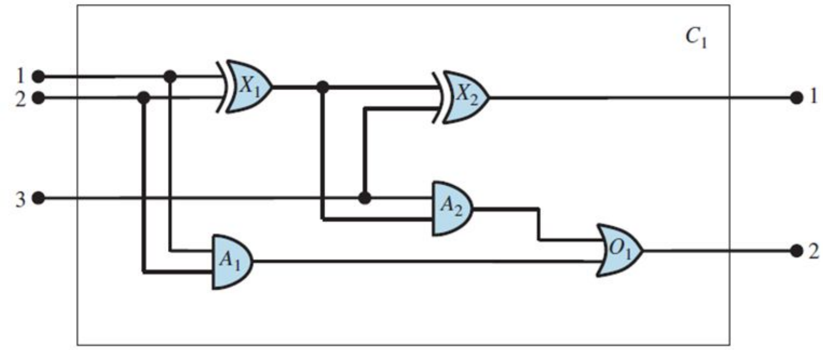
- Example:

  If two terminals are connected, then they have the same signal:

$$\forall\, t_1, t_2\ Terminal(t_1) \land Terminal(t_2) \land Connected(t_1, t_2) \Rightarrow Signal(t_1) = Signal(t_2)$$

Knowledge Engineering in FOL

Applications in the electronic circuits domain



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5. Encode the specific problem instance

- Categorize the circuit and its component gates & show the connections:

Connected(*Out*(1,X_1), *In*(1,X_2)) *Connected*(*In*(1,C_1); *In*(1,X_1))

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Knowledge Engineering in FOL

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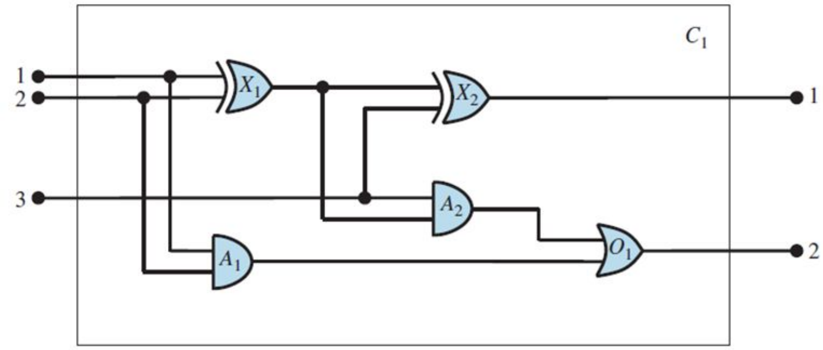

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6. Pose queries to the inference procedure

- What are the possible sets of values of all the terminals for the adder circuit?
- This final query will return a complete input–output table for the device, which can be used

Knowledge Engineering in FOL

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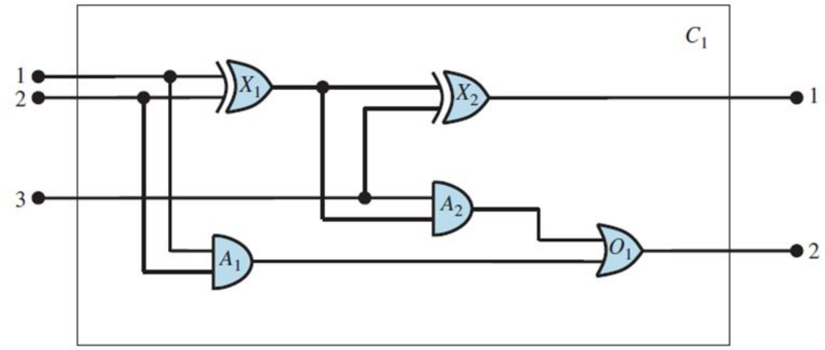

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7. Debug the knowledge base

- We can perturb the knowledge base in various ways to see what kinds of erroneous behaviors emerge
- Example if no assertion $1 \neq 0$

Summary

First-order logic:

– objects and relations are semantic primitives
– syntax: constants, functions, predicates, equality, quantifiers

Increased expressive power: sufficient to define

wumpus world

Situation calculus:

– conventions for describing actions and change in FOL
– can formulate planning as inference on a situation calculus KB

Developing a KB in FOL requires a careful process of analyzing the domain, choosing a vocabulary, and encoding the axioms required to support the desired inferences.

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