Dirk Bergemann
Joan Feigenbaum
Yale University

Economics and Computation

Computer Science 455/555 and Economics 425/563
Problem Set 5: Information, Recommendation and Reputation
11/20/08

This problem set is due on Tuesday, 12/02/08.

1. **Complementarity and Value of Access.** Consider a consumer with a utility function

$$u(q_1, q_2) = q_1 + q_2 + \theta q_1 q_2$$

where q_1 is the quantity/quality she purchases from firm 1 and q_2 the quantity/quality she purchases from firm 2. The parameter $\theta \in [0, 2]$ describes how much she values the complementarity between the products 1 and 2. Each firm i has a cost function $c_i(q_i) = \frac{1}{2}q_i^2$ of producing q_i. The net utility of the consumer is

$$q_1 + q_2 + \theta q_1 q_2 - p_1(q_1) - p_2(q_2)$$

and the net utility of the firm is

$$p_i(q_i) - \frac{1}{2}q_i^2.$$

In this first version of the problem we shall assume that there is complete information about θ and hence θ is common knowledge to the consumer and the firm (but naturally the solutions you will compute will depend on θ). We shall relax this assumption in the second version of the problem.

   1. Describe the socially optimal production plan for the firms to maximize the joint social surplus of the buyer and firms.
   2. Suppose that the products q_1 and q_2 are offered by a single firm. What is the quantity vector (q_1, q_2) and the price p for it that the single firm offers to maximized its profit so that the consumer is willing to participate in the trade, i.e. her net utility is larger or equal to zero. How does it compare to the socially efficient solution you computed above.

3. Suppose now that the two firms are acting separately and each can offer a quantity price pair so as to maximize its profit. Describe the symmetric Nash equilibrium of this game and compare it with the solution to (1.1) and (1.2). Are there asymmetric equilibria in this game as well?
4. Suppose now that the products are offered sequentially. Product 1 is offered in period 1, and product 2 is offered in period 2. The purchase of the product 1 is public information in period 2. Consider now the (subgame perfect) Nash equilibrium in which first firm 1 makes an offer, the buyer makes a decision of whether to accept or to reject, then firm 2 makes an offer which the consumer can again accept or reject. Compare the solution here with the solution to (1.1) and (1.3).
5. Finally, maintain the timing of (1.4), but suppose that firm 2 only gets in contact with the customer if firm 1 informs firm 2 about the customer (and after it has made a sales arrangement with respect to q_1 with the customer.) In addition suppose that firm 1 can charge firm 2 a fixed amount to pass the customer to firm 2. What are the quantities sold and what is the fee that firm 1 charges firm 2 to obtain access to the customer. How does the solution here compare with (1.1) and (1.4).

2. **Correlation and Recommender Systems.** Suppose there are two customers, $i = 1, 2$ and (two) copies of identical items (say books) to be sold by a single firm to the customer. The value of the item for each customer is

$$u_i(\theta) = \alpha\theta_0 + (1-\alpha)\theta_i$$

with a given and fixed $\alpha \in (0,1)$. The distribution of $\theta_0, \theta_1, \theta_2 \sim \mathcal{U}[-1,+1]$ but their value of is unknown to the firm and the customers. The parameter θ_0 represents a common component (quality), the parameter θ_i an idiosyncratic component (taste). After purchasing (and reading) the book, customer i observes his utility u_i, but does not observe the realization of either θ_0 or θ_i.

1. Suppose for the moment that the product is offered for free at a price $p = 0$.
    1. Suppose the customer can purchase the product only in a single period (and hence at the same time). What is the expected utility of the product for each consumer.
    2. Suppose now that customer 1 first purchases the product, observes his experience, reports it publicly, and then customer 2 has to make a decision as to whether he should get the product. Given the experience of the first customer, what is the conditional expected utility of the second customer. Compute it using Bayes rule for conditional expectation.

3. Compute the sum of the expected utility for the consumers in the sequential choice model. How does it compare to the simultaneous choice model.
4. Suppose $\alpha_i \neq \alpha_j$. What is the socially optimal order for the consumers to make their choice in the sequential model.

2. We maintain the model above except that we assume that $\theta_0 \sim \mathcal{U}[-1-\varepsilon, +1]$ for some small $\varepsilon > 0$. Suppose the cost of producing a copy of the product is $c = 0$ (think digital good) and that the firm is now charging the customers for the product.
   1. If the firm wants to sell the product simultaneously, what is the optimal symmetric price to charge and what is the decision of the consumers?
   2. Suppose now that the firm can offer the product over two periods, and that it can change the price across the two periods. Suppose also that the past customers post their experience publicly, so that later customers can observe their experience. What is the profit maximizing sequence of price for the firm to offer when the price in the second period can depend on the reported experience of the customer in the first period.

Readings. A very short introduction on recommender systems is given by (Resnick & Varian 1997). The lecture on recommender systems partially used material from (Karypis, Konstan & Riedl 2000) and (Herlocker, Konstan, Terveen & Riedl 2004). The economics of eliciting feed-back and the peer-prediction method is analyzed in (Miller, Resnick & Zeckhauser 2005). The economics of condition prices on past behavior is analyzed in (Acquisti & Varian 2005).

A very short introduction on reputation systems is given by (Resnick, Zeckhauser, Friedman & Kuwabara 2000). The lecture on reputation systems is based on Chapter 27 in (Nisan, Roughgarden, Tardos & Vazirani 2008) as well as (Friedman & Resnick 2001) and empirical evidence reported in (Resnick, Zeckhauser, Swanson & Lockwood 2006) and (Cabral & Hortascu 2008).

References

Acquisti, A. & H.R. Varian. 2005. "Conditioning Prices on Purchase History." *Marketing Science* 24:367–381.

Cabral, L. & A. Hortascu. 2008. "The Dynamics of Seller Reputation: Evidence from EBay." *Jounral of Industrial Economics* p. forthcoming.

Friedman, E. & P. Resnick. 2001. "The Social Cost of Cheap Pseudonyms." *Journal of Economics and Management Strategy* 10:173–199.

Herlocker, J., J. Konstan, L. Terveen & J. Riedl. 2004. "Evaluating Collaborative Filtering Recommender Systems." *ACM Transactions on Information Systems* 22:5–53.

Karypis, B. Sarwar Adn G., J. Konstan & J. Riedl. 2000. "Analysis of Recommendation Algorithms for E-Commerce." *Proceedings of the 2nd ACM Conference on Electronic Commmerce (EC00)* pp. 285–295.

Miller, N., P. Resnick & R. Zeckhauser. 2005. "Eliciting Informative Feedback: The Peer-Prediction Method." *Management Science* 51:1359–1373.

Nisan, N., T. Roughgarden, E. Tardos & V. Vazirani. 2008. *Algorithmic Game Theory.* Cambridge: Cambridge University Press.

Resnick, P. & H.R. Varian. 1997. "Recommender Systems." *Communications of the ACM* 40:56–58.

Resnick, P., R. Zeckhauser, E. Friedman & K. Kuwabara. 2000. "Reputation Systems." *Communications of the ACM* 43:45–48.

Resnick, P., R. Zeckhauser, J. Swanson & K. Lockwood. 2006. "The Value of Reputation on eBay: A Controlled Experiment." *Experimental Economics* 9:79–101.