ECON 425/563 // CPSC 455/555: Practice problems on information markets (Chapter 26 of AGT)

1. Consider Example 26.8 in AGT. Explain why, after the round-1 clearing price of \$0.75 is announced, both agents know that the OR of x_1 and x_2 is 1 and thus that the process converges to an equilibrium price of 1 after one more round.

   According to Theorem 26.12, the assumption in this example that the common prior distribution is uniform is unnecessary. Convince yourself that this is true by working through the same example with the following (nonuniform) common prior distribution: $P(x_1 = 0) = .25$, $P(x_2 = 0) = .75$, and the two inputs are statistically independent (*i.e.*, the joint distribution on (x_1, x_2) is just the product of the distributions on x_1 and x_2). Once again, assume that agent 1 observes $x_1 = 0$ and that agent 2 observes $x_2 = 1$.

2. Prove that the *n*-input AND function satisfies the hypothesis of Theorem 26.12, *i.e.*, that the Boolean function $f(\mathbf{x})$ that is 1 if and only if all x_i are 1 is a weighted threshold function.

   Work through the analog of Example 26.8 for the AND function. That is, assume that there are two agents, that the common prior distribution on (x_1, x_2) is uniform, that agent 1 observes $x_1 = 0$, and that agent 2 observes $x_2 = 1$. What does each agent bid in each round, and what is the clearing price in each round?

3. As seen in Example 26.10, the XOR function and the uniform common prior satisfy the conditions of Theorem 26.13. Give another example of a boolean function f that is not expressible as a weighted threshold function and a common prior for which the price of the security F does not converge to the value $f(\mathbf{x})$. By contrast, give an example of a common prior distribution for which the price of the security F *does* converge to the value of the XOR function.

4. In Section 26.2 of AGT, Pennock and Sami give several reasons that *no-trade theorems* do not necessarily model real-world traders' behavior, namely the dependence of these theorems on the assumptions of risk neutrality and common knowledge that all traders are completely rational Bayesians. Roughly speaking, these are "economic" explanations of why trade occurs despite these theorems. Give a "computational" explanation of the same phenomenon.