HW2, CPSC 468/568, Due Feb. 18, 2016

Throughout this assignment, if a proof or step of a proof follows directly from a definition given or a theorem proven in class or in a reading assignment, then you may simply say that, *i.e.*, you need not reproduce proofs given in class or in the reading.

Problem 1 *(20 points)*:

Prove that 2SAT is in NL.

Problem 2 *(15 points)*:

Let BIPARTITE be the set of all undirected graphs $G = (V, E)$ such that V is the disjoint union $V = V_1 \sqcup V_2$ of two vertex sets V_1 and V_2, and all edges in E have one endpoint in V_1 and one endpoint in V_2. Prove that BIPARTITE is in NL.

Problem 3(a) *(5 points)*:

Let $k \geq 1$ be a positive constant. Prove that NP $\not\subseteq$ DTIME(n^k).

Problem 3(b) *(15 points)*:

Prove that $\text{NP}^{\text{EXPCOM}} \not\subseteq \text{DTIME}^{\text{EXPCOM}}(n^k)$. Here, EXPCOM is the oracle used in class on Feb. 4, 2016, in the proof of the Baker-Gill-Solovay theorem, and $\text{DTIME}^{\text{A}}(n^k)$ is the class of sets recognizable by deterministic TMs that run in time $O(n^k)$ and have access to oracle A.

Problem 4 *(20 points)*:

Consider the complexity classes DTIME(n^2), NTIME(n^2), NSPACE(n^5), and DPSACE(n^8). For 4 points each, state and prove 5 containment relationships between pairs of these classes.

Problem 5 *(10 points)*:

Prove that NTIME(n^k) $\subsetneq$ PSPACE, for any constant $k \geq 1$. Does this imply that NP $\subsetneq$ PSPACE? Briefly justify your answer.

Problem 6 requires you to work through a proof of the Gap Theorem.

Let M_0, M_1, M_2, ..., be an enumeration of all Turing Machines, *e.g.*, the one in Figure 1.7 of your textbook. Let Γ_i be the tape alphabet of M_i.

For integers $i, k \geq 0$, define the following property $P(i, k)$: "Any machine among M_0, M_1, ..., M_i, on any input of length i, will halt in fewer than k steps, halt after more than 2^k steps, or not halt at all" – that is, on an input of length i, none of these machines will halt immediately after a number of steps that is in the interval $[k, 2^k]$.

Define $f(i)$, $i \geq 0$, as follows. Let $k_0 = 2i$, and, for $j > 0$, let $k_j = 2^{k_{j-1}} + 1$. We take $N(i)$ to be $\sum_{j=0}^{i} |\Gamma_j|^i$, *i.e.*, the total number of inputs of length i to the first $i + 1$ machines $M_0, \ldots, M_i$. Then $f(i)$ is defined to be k_ℓ, where ℓ is the largest integer less than or equal to $N(i)$ such that $P(i, k_\ell)$ is true.

Problem 6(a) *(5 points)*:

Prove that f is well defined, *i.e.*, that there must be an $\ell \leq N(i)$ such that $P(i, k_\ell)$ is true.

Problem 6(b) *(10 points)*:

Prove that any set in $\text{DTIME}(2^{f(n)})$ is in $\text{DTIME}(f(n))$.